This test is due on Monday, July 9th at 9:00 am. You may refer to your notes and textbooks, but you must not consult with other people. Please show all your work.

1. In the absence of predation, a population of fish grows according to the difference equation

$$\Delta P = rP(1-P)$$

where P is measured in units so that the carrying capacity is 1. Suppose fish are harvested at a rate proportional to the number of fish so that the new difference equation is

$$\Delta P = rP(1-P) - hP$$

where h is a proportionality constant indicating the degree of harvesting.

   (a) What is the biologically meaningful range of values for h?

   (b) Find the equilibrium points for the population with harvesting, expressing your answer in in terms of h and r. Indicate the stability of the equilibria.

   (c) Determine the harvesting level for which the only equilibrium is at $P = 0$.

2. For the following non-linear model

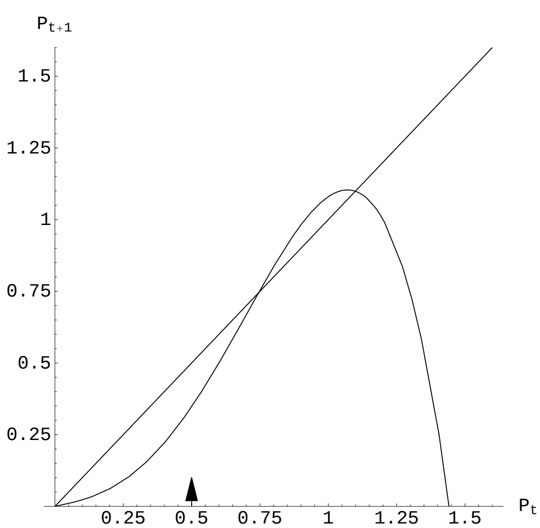
$$P_{t+1} = AP_t + B(P_t)^3$$

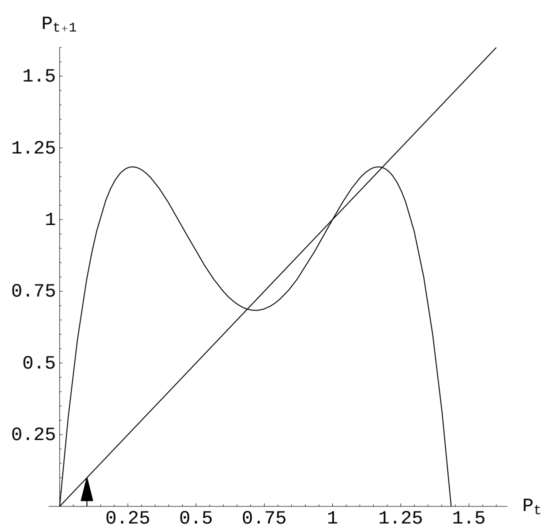
   (a) Write down an expression for the per capita growth rate.

   (b) What is the per capita growth rate at $P = 0$?

   (c) For what population is the per capita growth rate zero?

3. For each of the following graphs showing P_{t+1} vs P_t for some population label all the equilibrium points, indicate their stability, and draw a cobweb diagram showing the population for four time steps. starting at the point indicated





4. In a particular patch of forest there are only maple and fir trees. Suppose 1% of fir die each year and 4% of maple die each year. Whenever a tree dies a new tree grow in its place. Suppose that 80% of new trees are maple and 20% of new trees are fir. If we let f represent the number of firs and m represent the number of maples, write down a matrix equation for the growth of trees in the forest.

5. Given the following matrices,

$$A = \begin{pmatrix} 2 & -3 \\ 4 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & -2 \\ 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & -1 & 4 \\ 0 & 1 & 3 \end{pmatrix}, \quad D = \begin{pmatrix} 3 & 2 \\ 4 & -1 \\ 5 & 0 \end{pmatrix}$$

evaluate the following, without a calculator.

   (a) CD

   (b) B^3

   (c) A^{-1}

6. The following is the Leslie matrix for a certain animal population divided into two five year classes

$$M = \begin{pmatrix} 2/3 & 3/2 \\ 2/9 & 0 \end{pmatrix}$$

   (a) Interpret each of the numbers in the matrix.

   (b) Find the eigenvalues λ_1 and λ_2 and and the eigenvectors $\vec{v}_1$ and $\vec{v}_2$ for this matrix – show all your work.

   (c) If the initial population of the two classes is

$$\vec{x}_o = \begin{pmatrix} 100 \\ 100 \end{pmatrix}$$

   find the scalars c_1 and c_2 such that

$$\vec{x}_o = c_1\vec{v}_1 + c_2\vec{v}_2$$

   (d) Write down the general solution for $\vec{x}_t$ in terms of eigenvectors and eigenvalues.

   (e) Describe the long term behavior of the population. That is describe the steady state solution and sketch a graph of each population group as a function of time, clearly showing the transient behavior on your graph.