3.1 In 1672, an international effort was made to measure the parallax angle of Mars at the time of opposition, when it was closest to Earth; see Fig. 1.6.

(a) Consider two observers who are separated by a baseline equal to Earth’s diameter. If the difference in their measurements of Mars’ angular position is 33.6”, what is the distance between Earth and Mars at the time of opposition? Express your answer both in units of cm and AU.

(b) If the distance to Mars is to be measured to within 10%, how closely must the clocks used by the two observers be synchronized? Hint: Ignore the rotation of Earth. The average orbital velocities of Earth and Mars are 29.79 km s⁻¹ and 24.13 km s⁻¹, respectively.

\[ 2R_\oplus = d \theta \] where \( \theta = 33.6'' \)

\[ d = 2R_\oplus = 7.83 \times 10^{10} \text{ m} ] \text{ AU } = 5.3 \text{ AU] } \]

\[ v = 29.79 - 24.13 = 5.66 \times 10^3 \text{ km/s] } \]

\[ \Delta \bar{e} = \bar{d} = \frac{d^2 - d}{10} \text{ where the } \bar{d} \text{ is wrong, actually }\]

\[ \bar{d}' = \frac{2R_\oplus + \sqrt{vt}}{\theta} \]

\[ \theta = 1 - \frac{d}{d'} \]

\[ \frac{d'}{d} = 0.9 \Rightarrow \bar{d}' = \frac{10}{9} \bar{d} = \frac{10}{9} \left( 2R_\oplus \right) \]

\[ \bar{d}' = 2R_\oplus + \sqrt{vt} - \frac{10}{9} \left( 2R_\oplus \right) \Rightarrow 2R_\oplus \left( \frac{10}{9} - 1 \right) = \sqrt{vt} \]

\[ \sqrt{vt} = \frac{2R_\oplus}{\theta} = \frac{10}{9} \left( 2R_\oplus \right) \Rightarrow 2R_\oplus \left( \frac{10}{9} - 1 \right) = \sqrt{vt} \]

\[ \sqrt{vt} = \frac{2R_\oplus}{\theta} = \frac{10}{9} \left( 2R_\oplus \right) \Rightarrow 2R_\oplus \left( \frac{10}{9} - 1 \right) = \sqrt{vt} \]

\[ 2R_\oplus = \frac{6.38 \times 10^{10} \text{ m}}{9.166 \times 10^2 \text{ km/s}} = 250 \text{ s] } \]
3.6 A $1.2 \times 10^8$ kg spacecraft is launched from Earth and is to be accelerated radially away from the Sun using a circular solar sail. The initial acceleration of the spacecraft is to be $1g$. Assuming a flat sail, determine the radius of the sail if it is

(a) black, so it absorbs the Sun's light.
(b) shiny, so it reflects the Sun's light.

*Hint:* The spacecraft, like Earth, is orbiting the Sun. Should you include the Sun's gravity in your calculation?

\[ F = m \cdot a \quad \Rightarrow \quad F_{\text{rad}} = k \frac{\langle S \rangle A}{c} \]

\[ a = g = 9.8 \frac{m}{s^2} \]

\[ k = 3.1 \text{; absorption: BLACK} \]

\[ 2 \text{; reflection: SHINY} \]

\[ \langle S \rangle = \frac{E_0 B_0}{2 \mu_0} \text{; power = intensity of solar radiation at distance } d \]

\[ \text{Intensity } = \text{power} = \frac{10}{\text{area at distance } d} \]

\[ \text{Intensity} = \text{power} = \frac{10}{100 \text{ cm}^2} = 10 \text{ Watts} \]

\[ mg = k \frac{\langle S \rangle A}{c} \quad \Rightarrow \quad A = \frac{c \cdot mg}{k \langle S \rangle} \]

\[ A = 3.8 \times 10^8 \frac{m}{s} (1.2 \times 10^{11} \text{ kg}) 9.8 \frac{m}{s^2} \]

\[ k \cdot 1360 \text{ (Watts/s) = m/s} \quad \Rightarrow \quad \frac{w^2}{m^2} = \frac{w^2}{m^2} \]

\[ A = \frac{1}{k} \cdot 3.3 \times 10^{10} \frac{w^2}{m^2} = \pi R^2 \quad \Rightarrow \quad R = \sqrt{\frac{4}{\pi}} \]

\[ R_{\text{black}} = 10^5 \text{ m} = 100 \text{ km} \]

\[ R_{\text{shiny}} = \frac{1}{2} \pi \text{ km} = 72 \text{ km} \]

*Note that includes Sun's gravity, since sail is accelerating away from the Sun, not Earth.*
3.8 Consider a model of a star consisting of a spherical blackbody with a surface temperature of 28,000 K and a radius of 5.16 x 10^13 cm. Let this model star be located at a distance of 180 pc from Earth. Determine the following for the star:  

(a) Luminosity.  
(b) Absolute bolometric magnitude.  
(c) Apparent bolometric magnitude.  
(d) Distance modulus.  
(e) Radiant flux at the star's surface.  
(f) Radiant flux at Earth's surface (compare this with the solar con- 

\[ R = 5.16 \times 10^{13} \text{ m} \]

\[ L_0 = 3.826 \times 10^{33} \text{ erg/s} \left( \frac{1}{180} \right) \]

\[ = 3.826 \times 10^{26} \left( \frac{4}{5} \right) = 3.05 \times 10^{26} \text{ L_0} \]

\[ M = M_0 - \frac{5}{2} \log \left( \frac{r}{10} \right) \]

\[ = 7.6 - \frac{5}{2} \log \left( \frac{1.17 \times 10^{31}}{3.826 \times 10^{26}} \right) \]

\[ = -6.45 \] (magnitude is unitless)

\[ m = M + 5 \log \left( \frac{d}{10 \text{ pc}} \right) = -6.45 + 5 \log \left( \frac{180}{10} \right) \]

\[ m = -0.17 \]

\[ m-M = -0.17 + 6.45 = 6.28 \]

\[ \text{Flux} = \frac{\text{Power}}{\text{Area}} = \frac{L}{\pi r^2} = \frac{4}{3} \pi T^4 = 3.5 \times 10^{10} \text{ W/m}^2 \]

\[ \text{Flux/}F_0 = \left( \frac{T}{T_0} \right)^4 = \left( \frac{28000}{5770} \right)^4 = 1.1 \times 10^{15} \]

This is at each star's surface.
3.8: Ross's radiant flux at the Earth's surface

\[ F = \frac{L}{4\pi d^2} \]

where \( d = 180 \text{ pc} \)

\[ 3.1 \times 10^{16} \text{ m} = 5.16 \times 10^{18} \text{ m} \]

\[ F = 1.2 \times 10^{-3} \text{ W m}^2 \]

\[ \text{in} \left( 5.16 \times 10^{18} \text{ m}^2 \right) \]

\[ \frac{3.1 \times 10^{-8}}{4.7} \text{ W} \]

(3.15) \( \eta \) Peak wavelength using Wien's law: \( \lambda_{\text{max}} = 2.9 \times 10^{-3} \text{ km} \)

\[ \lambda_{\text{max}} = 2.9 \times 10^{-3} \text{ km} \]

\[ 10^4 \text{ km} \]

\[ 2.9 \times 10^{-4} \text{ m} \]

ultraviolet

3.13: Use the data in Appendix E to answer the following questions.

(a) Calculate the absolute and apparent visual magnitudes, \( M_V \) and \( V \), for the Sun.

(b) Determine the magnitudes \( M_B, B, M_U, \) and \( U \) for the Sun.

(c) Locate the Sun and Sirius on the color–color diagram in Fig. 3.10.

Refer to Example 3.6 for the data on Sirius.

\[ M_V = 4.83 \text{ absolute} \]

\[ 3.9 \text{ distance modulus} \]

\[ \mu - M = -31.57 \]

\[ U - B = 0.16 \]

\[ B - V = 0.64 \]

\[ S_0 \]

\[ B - V + 0.64 = -26.10 \]

\[ U = B + 0.16 = -25.94 \]

\[ M_U = U - \text{distance modulus} \]

\[ M_B = B - \text{distance modulus} \]

\[ M_V = -26.10 \]

\[ M_U = -25.94 \]

\[ M_B = -24.46 \]

\[ M_V = -26.10 \]

\[ M_U = -25.94 \]

\[ M_B = -24.46 \]

\[ +5.47 \]

\[ +5.63 \]