1. Verify equations (4.5) by writing out the components.

2. Let the position vector (with its tail at the origin) of a moving particle be 
\[ \mathbf{r} = \mathbf{r}(t) = t^2 \mathbf{i} - 2t \mathbf{j} + (t^2 + 2t) \mathbf{k}, \]
where \( t \) represents time.

(a) Show that the particle goes through the point \( (4, -4, 8) \). At what time does it do this?
(b) Find the velocity vector and the speed of the particle at time \( t \); at the time when it
passes through the point \( (4, -4, 8) \).
(c) Find the equations of the line tangent to the curve described by the particle and the
plane normal to this curve, at the point \( (4, -4, 8) \).

\[ \mathbf{r}_t = t^2 = 4 \quad \text{when} \quad t = 2 \]
\[ \mathbf{r}_y = -2t = -4 \quad \text{when} \quad t = 2 \]
\[ \mathbf{r}_z = t^2 + 2t = 8 \quad \text{when} \quad t = 2 \quad \checkmark \]

\[ \mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt} \left( t^2 \mathbf{i} - 2t \mathbf{j} + (t^2 + 2t) \mathbf{k} \right) \]
\[ \mathbf{v} = 2t \mathbf{i} - 2 \mathbf{j} + (2t + 2) \mathbf{k} \]

**Diagram:**

- \( r \) = curve described by particle
- \( \mathbf{v} \) = tangent to path = direction of normal plane (perpendicular)
- \( \mathbf{v} = 8 \) op of \( r \)

**Tangent Line:**
\[ \mathbf{P} + \mathbf{v} t = \mathbf{r}(2) + \mathbf{v}(2) t \]
\[ = (4, -4, 8) + (2t, -2, (2t^2 + 2t)) t \]
\[ = (4, -4, 8) + (2t, -2, (2t^2 + 2t)) \quad \checkmark \]
\[ = (4, -4, 8) + (4, -2, 6) t \]

**Tangent Line:**
\[ (4 + 4t) \mathbf{i} - (4 + 2t) \mathbf{j} + (8 + 6t) \mathbf{k} \]

\( \mathbf{N} = \mathbf{a} \mathbf{i} + \mathbf{b} \mathbf{j} + \mathbf{c} \mathbf{k} \) normal to a plane.
Booo (108) and \((\vec{r} - \vec{r}_0)\) is a vector in the plane,

then \(\vec{N}\) and \((\vec{r} - \vec{r}_0)\) are perpendicular, so the equation of the plane is \(\vec{N} \cdot (\vec{r} - \vec{r}_0) = 0\) or

\[ a(x-x_0) + b(y-y_0) + c(z-z_0) = 0 \]  \[ p.108 \] (5.10)

In our case, \((x_0, y_0, z_0) = (4, -4, 8) - P\)

and \(\vec{N} = \frac{\vec{V}_p}{\|\vec{V}_p\|} = (4, 2, 6) = a\hat{i} + b\hat{j} + c\hat{k}\)

So tangent plane equation is

\[ 4(x-4) - 2(y+4) + 6(z-8) = 0 \]

\[ 4x - 16 - 2y - 8 + 6z - 48 = 0 \]

\[ 2x - 8 - y + 4 + 3z - 24 = 0 \]

\[ 2x - y + 3z = 8 + 4 + 24 \]

\[ 2x - y + 3z = 36 \]