-11.7) Show that Planck's equation for the energy density of blackbody radiation reduces to the equation suggested by Wien for small values of $\lambda T$.

\[ u_{\text{pl}}(\lambda) = \frac{8\pi \hbar c}{\lambda^5} \left( e^{\frac{\hbar c}{kT\lambda}} - 1 \right) = \frac{a}{\lambda^5} e^{\frac{\hbar c}{kT\lambda}} - 1 \]

\[ b = \frac{hc}{kT} \]

\[ \text{Let } a = 8\pi \hbar c \]

(CM 29) Show that $\lim_{\lambda \to 0} u_{\text{pl}} = u_{\text{Wien}} = \frac{g_1 e^{-c_2/kT}}{\lambda^5}$

\[ \lim_{\lambda \to 0} (e^{\frac{\hbar c}{kT\lambda}} - 1) = e^{\frac{\hbar c}{kT\lambda}} \text{ which becomes much larger than 1.} \]

So $\lim_{\lambda \to 0} u_{\text{pl}} = \frac{g_1 e^{-c_2/kT}}{\lambda^5}$
Show that Planck's equation for the energy density of blackbody radiation reduces to the equation suggested by Rayleigh and Jeans for large values of $\lambda T$.

(Hint: Expand the exponential in a power series.)

\[ u = \frac{8\pi \hbar c}{15} \left( e^{\frac{\hbar}{kT}} - 1 \right) ^{-\frac{3}{2}} = \frac{a}{15} \left( e^{\frac{\hbar}{kT}} - 1 \right) ^{-\frac{3}{2}} \]

\[ \lim_{\lambda \to 0} u = \frac{8\pi \kappa_b T}{\lambda^4} \quad \text{(Rayleigh-Jeans)} \]

We derived the power series expansion for $e^{x}$:

\[ e^{x} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} \]

\[ (e^{\frac{\hbar}{kT}} - 1) = e^{x} - 1 = \left( 1 + x + \frac{x^2}{2} + \cdots + \frac{x^n}{n!} \right) - 1 = \frac{x + x^2 + \cdots}{2!} \]

where $x = \frac{\hbar c}{2kT}$. \[ \lim_{\lambda \to \infty} e^{\frac{\hbar}{kT}} = \]

Then \[ \lim_{\lambda \to \infty} u = \]
Show that Planck's equation predicts a maximum in the energy density of blackbody radiation at the point $\lambda = 0.290 \text{ cm K/T}$ when $u(T, \lambda)$ is plotted against $\lambda$. [Hint: You may solve the problem either by a one-dimensional grid search or by iterative methods.]

$$u = \frac{q}{\lambda^5 \left( e^{\frac{q}{kT}} - 1 \right)}$$

$$a = \frac{8\pi}{k c}, \quad b = \frac{u}{k T}$$

Extreme in $u$ when $\frac{\partial u}{\partial a} = 0$

Differentiate, set to 0,

$$\frac{\partial u}{\partial a} = \frac{9}{8 \pi} \frac{2}{\lambda^3} \left( \frac{1}{e^{\frac{q}{kT}} - 1} \right) + \left( e^{\frac{q}{kT}} - 1 \right) \frac{2}{\lambda^3} \frac{\partial q}{\partial a}$$

Solve for $\lambda$:

$$\frac{1}{\partial \lambda} f^p(\lambda) = p f^p(\lambda) \frac{\partial}{\partial \lambda} f(\lambda), \quad \text{let } f(\lambda) = (e^{\frac{q}{kT}} - 1)^{-1}$$

$$\frac{2}{\partial \lambda} (e^{\frac{q}{kT}} - 1)^{-1} =$$