Finishing modern physics - week 8

Angular momentum, magnetic moment, Zeeman effect

**Bohr Atom**

\[ F = ma \]

\[ \frac{kqQ - \frac{e^2}{r^2}}{r} = \frac{mv^2}{r} \rightarrow \text{Solve for } v \]

Current \( I = \frac{dq}{dt} = \frac{e}{\text{period}} \)

\[ \text{period} = T = \frac{\text{circumference}}{v} = \frac{2\pi r}{v} \]

\[ J = \frac{ev}{2\pi r} \]

Area of e⁻ orbit \( A = \pi r^2 \)

**Magnetic Moment** \( \mu = IA = \frac{ev}{2\pi r} \pi r^2 = \frac{evr}{2} \)

\[ \text{Write this using Angular Momentum} \quad L = \mu vr \rightarrow vr = L \]

\[ \text{Solve for } vr, \quad \text{plug into } \mu = \frac{eL}{2me} \]

**Quantization** of orbital angular momentum: \( L = \hbar n \)

Write \( \mu \) in terms of \( m \): \( \mu = \frac{e}{2} \frac{\hbar}{m} \)

\( \text{Bohr Magneton } \mu_B = \frac{e\hbar}{2me} \)
Bohr atom: electron's orbit → magnetic dipole 

(DRAW B FIELD)

Magnetic dipole experiences TORQUE in a magnetic field \( \vec{B} \)

Potential energy \( U = -\vec{\mu} \cdot \vec{B} \)

Zeeman effect: energy levels SPLIT by \( \Delta U = \mu_0 B \)

\[ \frac{2\hbar}{1005} \text{ Orbital angular momentum QUANTUM NUMBER } l = 0, 1, \ldots, (n-1) \]

Magnetic quantum number \( m_l \) indicates DIRECTION of \( \vec{L} \)

\[ \text{Orbital angular momentum } \vec{L} = \sqrt{l(l+1)} \vec{l} \]

For \( l = 2 \), \( L_z = m_l \hat{z} \)

If \( l = 2 \), then \( m_l \) can be 0, ±1, or ±2.

\[ m_l = 0, \pm 1, \pm 2 \]

Do 46spdf

MNEMONIC for

Filling order:

\[ \text{spdf} \]

0 1 2 3

8s, p1, po, d2, pf, so
45. (I) Verify that the Bohr magneton has the value
\[ \mu_0 = 9.27 \times 10^{-24} \text{ J/T} \] (see Eq. 40-12).

46. (II) Suppose that the splitting of energy levels shown in
Fig. 40-4 was produced by a 2.0-T magnetic field. (a) What
is the separation in energy between adjacent \( m_l \) levels for
the same \( \ell \)? (b) How many different wavelengths will there
be for 3d to 2p transitions, if \( m_l \) can change only by \( \pm 1 \) or
0? (c) What is the wavelength for each of these transitions?

\[ U = -\mu_0 B = -\mu_0 B \mu_0 \]
\[ \Delta U = \Delta m \mu_0 B = 1.2 \times 10^{-4} \text{ eV} \]

Transitions are allowed from \( \Delta m = 0, \pm 1 \)

\[ \frac{1}{n_0} = R \left[ \frac{1}{u^2} - \frac{1}{u_1^2} \right] = 1.0974 \times 10^{-7} \text{ m} \left( \frac{1}{4} - \frac{1}{9} \right) \]
\[ n = 2, \quad n' = 3 \]
\[ \lambda_0 = 656.10 \text{ nm} \]

\[ E = \frac{\hbar c}{\lambda} \]
\[ \frac{dE}{d\lambda} = \frac{\hbar c}{\lambda^2} \]

\[ \frac{dE}{\lambda} = -\frac{\hbar c}{\lambda^2} \]
\[ \Delta E = \frac{\hbar c}{\lambda} \Delta \lambda = \mu_0 B = 9.27 \times 10^{-24} \text{ J/T} \]
\[ \Delta E = 1.85 \times 10^{-23} \text{ J} = 1.16 \times 10^{-19} \text{ eV} \]

\[ E_0 = \frac{\hbar c}{\lambda_0} = 6.63 \times 10^{-34} \text{ J s} \times 3 \times 10^8 \text{ m/s} = 3.03 \times 10^{-19} \text{ J m} = 1.85 \text{ eV} \]

\[ E_0 + \Delta E = E_+ = \frac{\hbar c}{\lambda_0} = \frac{\hbar c}{\lambda} \]
\[ \lambda_0 = \lambda_0 - \Delta \lambda = \lambda = (1 - \frac{\lambda_0}{\lambda}) \lambda_0 \]

\[ \lambda = (1 - \frac{\lambda_0}{\lambda}) \lambda_0 = 656.14 \text{ nm} \]

\[ \lambda_0 - \Delta \lambda = \lambda_0 \]

\[ \lambda = (1 - \frac{\lambda_0}{\lambda}) \lambda_0 = 656.06 \text{ nm} \]
Heisenberg Uncertainty Principle

Uncertainty in position \( \Delta x \approx \lambda \) of light used to look at particle.

Photon can change particle's momentum by \( sp \approx \frac{\hbar}{\lambda} \)

\( \Delta x \Delta p \approx \frac{\hbar}{2\pi} \)

\( 2\pi \Delta x \Delta p \approx \hbar \)

Distance \( D \) can be measured by \( \frac{\hbar}{2\pi D} \)

The more precisely we measure position (with light of small \( \lambda \)), the less precisely we can know momentum.

Time uncertainty \( st \approx \frac{\Delta x}{c} \approx \frac{\lambda}{c} \) (travel time of photon across object)

Photon can change particle's energy by \( SE \approx \frac{\hbar c}{\lambda} \)

\( SEst \approx \frac{\hbar}{2\pi} \)

\( SEst \approx \frac{\hbar}{2\pi} \)