
This is a CLOSED-BOOK exam to be taken in class. This is designed as a two-hour exam. You have 3 hours. Pace yourself, and take breaks when you need them. SHOW YOUR WORK, to receive full credit, and include units. Please circle or underline your answers when appropriate, for clarity. Keep answers in simplest exact form or make order-of-magnitude estimates.

(sign legibly) ______________ Zita’s solutions ____________________

I affirm that I have worked this exam with WITHOUT using a calculator, text, HW, quizzes, computer, classmates, or other resources.

Everyone do the whole exam – except the last page. Part VI Astrophysics is optional, for Phys-B students who want to show improved understanding since the midterm.

Possibly useless information (please ask if you need more info)

\[ G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2 \quad \text{g} = 9.8 \text{ m/s}^2 \quad \text{parsec} \sim 3 \text{ ly} \quad c = 3 \times 10^8 \text{ m/s} \quad h = 6.63 \times 10^{-34} \text{ J.s} \]

\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N.m}^2 \quad \sigma = 5.67 \times 10^{-8} \text{ W/m}^2.\text{K}^4 \quad k = 1.38 \times 10^{-23} \text{ J/K} \]

\[ F = mg \quad F = -GmM/r^2 \quad F = -qQ/(4\pi\varepsilon_0 r^2) \quad F = -kx \quad F = -dU/dx \]

\[ F = dp/dt = ma \quad p = mv \quad s = r\theta \quad v = r\omega \quad a = r\alpha \]

\[ T = dL/dt = I\alpha \quad L = I\omega = mv r \quad \omega = d\theta/dt \quad \alpha = d\omega/dt \]

\[ K = \frac{1}{2} I\omega^2 \quad K = \frac{1}{2} mv^2 \quad I = \Sigma m_i r_i^2 \quad \lambda(\text{m})T(\text{K}) \sim 3 \times 10^{-3} \quad F = \sigma T^4 = L/\text{area} \]

\[ I = dq/dt \quad V = IR \quad P = IV \quad W = qV \quad E = -dV/dx \quad F = qvxB = ILxB \]

\[ p = 1/d \quad pc \sim 3 \times 10^{18} \text{ m} \quad m-M = 5 \log (d/10 \text{ pc}) \quad M - M_{\text{sun}} = 5/2 \]

\[ \log(L_{\text{sun}}/L) \]

\[ M_{\text{sun}} = 4.76 \quad L_{\text{sun}} = 3.826 \times 10^{33} \text{ erg/s} \]

<table>
<thead>
<tr>
<th>Sun</th>
<th>Earth</th>
<th>Moon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius</td>
<td>7 \times 10^5 \text{ km}</td>
<td>6.4 \times 10^3 \text{ km}</td>
</tr>
<tr>
<td>Mass</td>
<td>2 \times 10^{30} \text{ kg}</td>
<td>6 \times 10^{24} \text{ kg}</td>
</tr>
<tr>
<td>Distance between them: 1.5 \times 10^8 \text{ km}</td>
<td>3.84 \times 10^5 \text{ km}</td>
<td></td>
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</tbody>
</table>

___ / 13 Part I – Calculus
___ / 12 Part II – Research
___ / 30 Part III – Universe
___ / 21+14 Part IV – Electric and Magnetic fields
___ / 15 + 17 Part V – Electromagnetism
___ / 122 total
___ / 17 Part VI – Astrophysics extra credit
\[ \frac{d}{dx} A \cos(kx-\omega t) = -k A \sin (kx-\omega t) \]

\[ \frac{d}{dt} A \sin(kx-\omega t) = -\omega A \cos (kx-\omega t) \]

\[ \frac{d}{dy} \sin^2(y^3) = 2 \sin(y^3) \cos(y^3) \cdot 3y^2 = 6y^2 \sin(y^3) \cos(y^3) \]

\[ \frac{d}{dx} e^{-ax^2} = -2ax e^{-ax^2} \]

\[ \frac{d}{dx} (bx)^3 e^{-ax^2} = e^{-ax^2} \left[ 3b^3x^2 - 2ax^2 (bx)^3 \right] = e^{-ax^2} \left[ 3b^3x^2 - 2ab^3x^4 \right] \]

\[ \frac{d}{dx} e^{-ax^2} \sin kx = e^{-ax^2} \left[ k \cos kx - 2ax \sin kx \right] \]

\[ \int 10ab \, dx = 10abx + C \]

\[ \int 3 \cos(4x) \, dx = \frac{3}{4} \sin 4x + C \]

\[ \int xe^x \, dx = xe^x + C \]

\[ \int (ax^2 + bx + c) \, dx = \frac{a}{3}x^3 + \frac{b}{2}y^2 + cx + d \]

\[ \int \left( \frac{a}{x^2} + \frac{b}{x} \right) \, dx = 2 \ln |x| + \frac{a}{x} + b \ln |x| + C \]

\[ \int A \cos(kx-\omega t) \, dt = -\frac{A}{\omega} \sin (kx-\omega t) + C \]

\[ \int A \sin(kx-\omega t) \, dx = -\frac{A}{k} \cos (kx-\omega t) + C \]
Part II - RESEARCH QUESTIONS (5-10 minutes)

1. A cyclotron uses an alternating voltage to
   (a) bend charged particles into a circular path
   (b) accelerate particles in a linear path
   (c) **accelerate particles in a circular path**
   (d) trap particles electrostatically in a vacuum chamber

2. A cyclotron without an alternating voltage can be used as
   (a) an accelerator
   (b) a toaster
   (c) a vacuum chamber
   (d) **a mass spectrometer**

3. Signals from particles of different masses can be distinguished in a cyclotron using the mathematical technique of
   (a) Integration by parts
   (b) Partial differentiation
   (c) **Fourier transformation**
   (d) Renormalization

4. **Why would we assume the Moon's spin angular momentum changes** when its orbital angular momentum changes?
   a) **Tidal locking is a stable situation:** since the Moon is already tidally locked with the Earth, the Moon’s the spin angular momentum will tend to compensate for any change in the Moon's orbital angular momentum.
   b) The reaction forces on the Moon (caused by tidal interactions) directly affect the Moon's spin, not its orbit.
   c) It would not; the spin angular momentum and orbital angular momentum are completely unrelated when cosidering the Earth-Moon as a closed system.

5. **Why can we ignore the Earth's orbital angular momentum when examining the angular momentum of the Earth-Moon system?**
   a) We can't; the Moon directly affects the orbital angular momentum of the Earth.
   b) **The Earth's orbit around the Sun is not affected by the Moon:** thus when cosidering the Earth-Moon system as a closed system, the orbital angular momentum of the Earth remains constant, and can essentially be ignored.
   c) Because we can assume that the Earth's orbital angular momentum will compensate for changes in Earth's spin angular momentum (in the same way that the Moon's does) due to tidal locking

6. **What is not a primary function of a spark plug?**
   A.) Ignite the air fuel mixture
   B.) Remove Heat from the combustion chamber
   C.) **Clean the engine**
7. As the spark gap increases, the voltage required to ignite the air-fuel mixture
   A.) remains constant
   B.) increases
   C.) decreases

8. Consider a standing wave propagating perpendicularly between two reflective surfaces. What might be a possible relation between its wavelength and the distance between the surfaces? (Hint: draw it) $N=$integer.
   a. $0.5N \times$wavelength = distance
   b. $1.5 \times$wavelength = distance
   c. both (a) and (b). are possible.

9. What are the components of an LRC circuit?
   a. lettuce rice and cantaloupe
   b. inductor resistor and a capacitor
   c. two resistors and a frequency generator
   d. a coil of wire wrapped around a metal core near another coil of wire

10. In Charles Wilson's model of Auroral Infrasonic Wave generation, the supersonic motion of the auroral surge is caused by:
    a. the Strong force
    b. the Lorentz force
    c. an electromotive force
    d. the Coriolis force

11. In Maxwell’s model, auroral infrasound is generated by
    a. Magnetic waves changing character near beta=1 surfaces
    b. Magnetic waves heating the atmosphere
    c. Static electricity in the ionosphere
    d. An intelligent designer

12. Infrasound is:
    a. a higher frequency than most people can hear, above 20 kHz
    b. a lower frequency than most people can hear, below 20 Hz
    c. both a) and b)
    d. a myth perpetuated by conspiracy theorists
1. According to the inverse-square law, if two stars have the same luminosity and if one star is 10 times farther away than the other, then
   - the more distant one would be 100 times fainter.
   - the more distant one would be 10 times fainter.
   - the closer one would be 100 times fainter.
   - they would look the same.

2. The color of a nearby but isolated star appears to be redder than that of the Sun. Which of the following conclusions is the most likely?
   - The star is moving very rapidly away from the Sun.
   - The star's temperature is lower than that of the Sun.
   - Dust and gas between the star and Earth has absorbed or scattered much of the blue light of the star.
   - Our night vision is more sensitive to red light.

3. How does a main-sequence star's lifetime depend on its overall mass?
   - The star's lifetime is almost independent of its mass because the higher hydrogen fuel "burning" rate of the more massive star is offset by the larger amount of fuel available.
   - The higher the star's mass, the shorter its lifetime because a more massive star "burns" hydrogen fuel much faster than a low-mass star.
   - The higher the star's mass, the longer the star's lifetime because there is more fuel available for thermonuclear "burning."

4. What physical process generates a force inside a pre-main-sequence star to offset the force of gravity and stop the star from slowly condensing and shrinking, thus producing a stable, nonshrinking main-sequence star?
   - Degeneracy pressure is generated by electrons when they are forced very close together because of the quantum-mechanical Pauli Exclusion Principle.
   - No physical process can prevent this condensation until a black hole is produced at the center of the star. Contraction of a star continues slowly throughout its lifetime.
   - Nuclear fusion begins as the temperature rises and this generates additional heat that produces an increase in internal gas pressure.

5. A certain star is seen to have a relatively low surface temperature but a very high luminosity. What can we conclude from these observations?
   - The star must be very large.
   - The star is a main sequence star, about the size of the Sun.
   - The star is a brown dwarf.

6. Explain your reasoning.
   \[ L \sim R^2 T^4 \]
7. Cepheid variable stars are important because we can
- find the distance to the Cepheid star after measuring only its period and apparent magnitude.
- find the distance to the Cepheid star after measuring only its period and absolute magnitude.
- find the absolute magnitude after measuring only its apparent magnitude.
- communicate with extraterrestrials using the stars’ natural frequencies.

8. What causes a Type Ia supernova?
- In a binary star system, a giant star filling its Roche lobe dumps gas onto a white dwarf, putting the white dwarf over the Chandrasekhar mass limit.
- Helium shell flashes push the outer layers of an asymptotic giant branch star out into space, leaving behind a stellar remnant.
- The core of a massive star is gradually transformed into iron by thermonuclear fusion reactions until it collapses, causing the outer layers to explode from the star.

9. Type Ia supernovae are important because
- They are the best supernovae for recycling metals into the universe, for incorporation into next-generation stars.
- They are the best supernovae for driving shock waves in their environments, stimulating new star formation.
- They are the supernovae with the best known luminosity, capable of serving as standard candles for distance measurements.

10. Which of the following will NOT occur as an isolated star evolves?
- The star gains mass because nuclear fusion reactions convert lighter elements into heavier elements.
- The star loses mass to space.
- The chemical composition of the star changes as nuclear fusion reactions convert lighter elements into heavier elements.

11. Why do nuclear fusion reactions which produce heavier elements than iron NOT produce thermonuclear energy to maintain the luminous output of a high-mass star late in its life?
- No known nuclear reactions occur between iron and other nuclear particles because of the structure and extraordinary stability of these nuclei.
- It takes extra energy to add further protons to iron nuclei, rather than producing energy as it does in fusion of lighter nuclei.
- Electron degeneracy prevents protons from approaching iron nuclei in the core of the star.

12. Who made the first discovery of a pulsar?
- Fritz Zwicky and Walter Baade
- Albert Einstein
- Jocelyn Bell
13. What produces the rapid rotation rate of a young neutron star, or pulsar?
- The core of the dying star spins up because it collapses to a very small radius.
- Matter falling onto the neutron star from the debris of the supernova explosion causes the neutron star to spin up.
- Mass transfer from a companion star causes the neutron star to spin up.

14. What is the force that keeps a neutron star from collapsing to a black hole under its intense self-gravitational field?
- Neutron degeneracy pressure, the quantum-mechanical effect in which no two neutrons with the same properties can occupy the same space.
- The intense nuclear repulsion between neutrons, only felt when these neutrons are very closely packed because the nuclear force is very short-ranged.
- The very high temperature and velocity of the neutrons, which creates a thermal gas pressure to oppose gravity.
- The centrifugal force of the star’s very rapid rotation rate.

15. What would happen to the Earth in its orbit if the Sun were to be replaced by a black hole of 1 solar mass on the stroke of midnight tonight?
- The Earth would begin to move in a straight line at a tangent to its previous orbit.
- The Earth would begin to spiral inward under the gravitational force of the black hole.
- The Earth’s orbit would not change.

16. Gravitational redshift is due to
- the Doppler shift when gravitating objects make other objects fall toward them, and thus away from the observer.
- the inability of light or matter to escape from inside the event horizon of a black hole.
- the slowing down of time in a gravitational field.

17. What causes the cosmological redshift of light?
- The wavelength of photons from a distant galaxy becomes stretched by the Doppler shift because of the motion of our Galaxy through expanding space.
- As photons leave a distant galaxy, their wavelength becomes stretched by the Doppler shift because of the galaxy's motion through space relative to us.
- The wavelength of photons from a distant galaxy becomes stretched by the expansion of space while the photons are traveling toward us.

18. Galactic radio emission at the wavelength of 21 cm is produced by
- electron transitions from the level \( n = 100 \) to \( n = 99 \) in atomic hydrogen atoms.
- the rearrangement of the "spins" of proton and electron from parallel (spin axes aligned) to antiparallel (spin axes opposite in direction) in atoms of atomic hydrogen.
- Electrons recombining with protons to form neutral atomic hydrogen, where the electron falls from outside the atom into the \( n = 21 \) level.
19. The fact that the orbital speeds of material moving around the galactic center appear to be constant across the Galaxy indicates that
- this motion is dominated by the supermassive black hole in the center of the Galaxy.
- most of the mass of our Galaxy is in the form of dark matter that emits no electromagnetic radiation.
- most of mass of our Galaxy is in the observed stars and interstellar dust and gas.

20. Because nothing, not even light, can escape from a black hole, how has the existence of such an object been established?
- By the extreme gravitational redshift of light emitted by a close companion star.
- By the observation of its gravitational effect upon a visible companion in a binary star.
- By the existence of a very dark area in the sky, from which nothing is being emitted.
- By divine revelation.

21. The radius of the event horizon of a black hole, the Schwarzschild radius, is constant, as predicted by the general theory of relativity.
- will be smaller the greater the mass of the black hole, because matter will then be more condensed.
- will be larger the greater the mass of the black hole.

22. What physical relationship has been used to estimate the total amount of mass within a few light-years of the galactic center?
- Kepler's third law.
- The period-luminosity relation.
- Newton's third law.
- Murphy’s law

23. The characteristics of the radio source Sagittarius A*, which indicate that this might be a supermassive black hole, are
- extreme gravitationally redshifted light in stars near this object, indicative of a large mass, and the appearance of a small black space at its precise position.
- very rapid inward motion of many nearby stars, indicative of a massive attractor, and very short IR bursts, indicative of heating of matter as it disappeared into the black hole.
- very high-speed motions of stars in elliptical orbits around this object, indicating a large concentration of mass, and very short X-ray flares, indicative of small physical size.

24. Suppose it was discovered that Hubble's constant, $H_0$, was smaller than previously thought. How would this affect our estimate of the age of the universe?
- It would have no effect on our estimate of the age of the universe.
- The age of the universe would be smaller.
- The age of the universe would be larger.
25. Scientists have recently been able to **estimate the degree of "flatness" of the universe**, which determines whether we live in an open or a closed universe, by measuring
- the **typical sizes of brighter structures in the cosmic microwave background**.
- the ratio of the average density of matter to that of radiation energy in the universe as a whole.
- the "lensing" or the bending of light from distant galaxies.

26. **We can see only a certain distance out into the universe, and this distance is the same in all directions. Why?**
- We are at the center of the universe.
- **The universe has a definite age.**
- The universe is not really expanding, but light loses energy as it travels, and photons fade out after a certain distance.

27. **Which of the following scenarios for our universe is most likely, based on recent results from very bright and very distant type Ia supernovae?**
- We live in a "flat" universe with a decelerating rate of expansion.
- We live in an open universe with a constant rate of expansion given by Hubble’s law.
- **We live in a "flat" universe with an accelerating rate of expansion.**

28. **The recently estimated density of radiation and matter in the universe (including dark matter) appears to be only about 30% of the critical density required for a flat universe. However, measurements of structure in the cosmic microwave background radiation have recently indicated that, in fact, we do live in a flat universe, intermediate in density between a closed and an open universe. What form is the additional 70% of the "matter," needed to reconcile these discordant results, likely to take?**
- Neutrinos, which are very abundant even if they have very little rest mass and are very difficult to detect.
- **Dark energy**, which generates no gravitational effects and emits no radiation.
- Antimatter, which generates a negative gravitational effect and will be detected only if it meets matter and is annihilated to produce radiation.

29. **The cosmic background radiation is almost perfectly uniform over the whole sky. Why was this a problem for the Big Bang theory?**
- Our Sun and our Galaxy are moving through space, so the Doppler shift due to these motions should make the part of the sky toward which we are moving appear slightly hotter and the opposite side slightly cooler.
- The Big Bang was an explosion, and therefore should have been "lumpy." This would produce hotter and cooler regions around the sky, so the background radiation should not be smooth.
- **Opposite sides of the observable universe are too far apart to have exchanged heat during the lifetime of the universe** to produce uniform temperature.

30. **How is this problem resolved, theoretically?**
Inflation caused regions of space initially in close contact to expand rapidly apart (at v>c)
Part: Electrostatics and magnetism: (30 minutes)

1. What causes electric fields? Name at least two very different phenomena.
   \[ \text{Charges (Gauss' law) and changing magnetic fields} \]
   \[ \text{Faraday's law} \]
   \[ \nabla \cdot \mathbf{E} = \frac{\partial \mathbf{D}}{\partial t} \]

2. What causes magnetic fields? Name at least three very different phenomena.
   \[ \text{Currents, permanent magnets (aligned domains of electron spin)} \]
   \[ \text{and changing} \ E \ \text{fields: Ampere's law} \]
   \[ \oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I + \mu_0 \epsilon_0 \nabla \times \mathbf{E} \cdot d\mathbf{A} \]

3. How do electric fields affect charges? Give a very specific example with a drawing.
   \[ \mathbf{F} = q \mathbf{E} \]
   \[ \text{accelerated in parallel to the} \ E \ \text{field} \]
   \[
   \begin{array}{c}
   \mathbf{E} \\
   \theta
   \end{array}
   \]

4. How do magnetic fields affect charges? Give a very specific example with a drawing.
   \[ \mathbf{F}_\perp = q \mathbf{v} \times \mathbf{B} \]
   \[ \text{perpendicular force} \]
   \[ \text{no change in speed} \]

5. How can you tell if a charge's motion is caused by an electric or magnetic force?
   \[ \text{If it BENDS, there is a magnetic field or} \ E \perp \mathbf{V} \]
   \[ \text{If it ACCELERATES, there is an electric field.} \]
   \[ \text{Combined case of} \ E \parallel \mathbf{B} \text{can cause more complex (or simple) motion} \]

6. Check all the fields that could possibly be present, for each charge motion.
   
   a. Charge at rest
   \[ \checkmark \ E \ only \]
   \[ \checkmark \ B \ only \]
   \[ \checkmark \ both \ E \ and \ B \]
   
   b. Charge moving at a constant speed
   \[ \checkmark \]
   
   c. Charge accelerating in a straight line
   \[ \checkmark \]
   
   d. Charge bending at constant speed
   \[ \checkmark \]
   
   e. Charge bending and accelerating
   \[ \checkmark \]

7. Describe how equipotentials are related to electric fields, (a) qualitatively and (b) quantitatively.
   \[ E = -\nabla V = -\frac{\partial V}{\partial x} \]
   \[ \nabla V = \int E \cdot dx \]
   \[ \text{Equipotentials are} \perp \text{to} \ E \ \text{and parallel to conductors (in electrostatics)} \]

8. \[ \text{Diagram} \]
8. **Equipotential lines**
   (a) are always parallel to electric field lines
   (b) are always perpendicular to electric field lines
   (c) may curve at different angles to electric field lines

9. Is an electric field stronger where equipotentials are concentrated or spread out? Explain.
   \[ E = \frac{\Delta V}{\Delta x} = {\text{gradient in } V} \]

10. Is an electric field stronger near a pointy object or a smooth one? Explain.
    
11. Is it safer to stand up or crouch down in a lightning storm? Explain.
    
12. Draw a diagram to illustrate your reasoning above.

13. Say a 2-ampere current flows for 1 minute from a 10-volt power supply.

   a. How much charge passes through the circuit?
   \[ I = \frac{Q}{t} \implies Q = It = 2 {\text{amp}} \times 60 \text{sec} = 120 \text{ coulomb} \]

   b. What is the resistance of the circuit?
   \[ V = IR \implies R = \frac{V}{I} = \frac{10 \text{ volts}}{2 \text{ amp}} = 5 \text{ ohms} \]

   c. How much power does the circuit use?
   \[ P = IV = 2 \text{ amp} \times 10 \text{ volts} = 20 \text{ watts} \]

   d. How much energy does the circuit use?
   \[ P = \frac{\text{energy}}{\text{time}} \implies E = Pt = 20 \text{ Joules} \times 60 \text{ sec} = 1200 \text{ Joules} \]

   e. How could you decrease the current through this resistor without changing the dial on the power supply? Would you add a resistor in parallel or series to the circuit? A bigger resistor or a smaller one, or does it matter? The bigger, the better.

\[ I_2 = \frac{V}{R_1 + R_2} \leq I_1 \]

\[ \text{Same current through both resistors.} \]
1. Find the direction of the magnetic force for each situation below: **ASSUME POSITIVE CHARGES**

   (a) \[ \mathbf{v} \times \mathbf{B} = F \]
   (b) \[ \mathbf{v} \times \mathbf{B} = F \]
   (c) \[ \mathbf{v} \times \mathbf{B} = F \]
   (d) \[ \mathbf{v} \times \mathbf{B} = F = 0 \]
   (e) \[ \mathbf{v} \times \mathbf{B} = F \]
   (f) \[ \mathbf{v} \times \mathbf{B} = F \]

2. Derive the cyclotron frequency from Newton's second law with the magnetic Lorentz force.

\[
\begin{align*}
F &= \mathbf{q} \mathbf{v} \times \mathbf{B} \\
q \mathbf{v} \mathbf{B} &= \frac{\mathbf{q} \mathbf{v}^2}{r} \\
\frac{qB}{\mu} &= \frac{v}{r} = \omega
\end{align*}
\]

3. A proton and an electron have the same kinetic energy upon entering a region of constant magnetic field.

(a) **How does the magnetic field change the kinetic energy** of each particle? Explain.

The magnetic field does no work since \[ \mathbf{F} = q \mathbf{v} \times \mathbf{B} \] is perpendicular to displacement \[ \mathbf{x} \times \mathbf{v} \] and \[ \Delta W = \int \mathbf{F} \cdot d\mathbf{x} = 0 = \Delta KE \]

(b) **Derive the ratio of the radii** of their circular paths analytically – that is, in simplest form, without numbers. (Find \( r_\text{e}/r_\text{p} \).)

\[
\begin{align*}
\frac{1}{2} m_\text{e} v_\text{e}^2 &= \frac{1}{2} m_\text{p} v_\text{p}^2 = K \quad \text{and} \quad r = \frac{v}{q B} \\
\frac{r_\text{e}}{r_\text{p}} &= \frac{m_\text{e} v_\text{e}}{q B} = \frac{m_\text{e} v_\text{e}}{m_\text{p} v_\text{p}} \\
\left( \frac{r_\text{e}}{r_\text{p}} \right)^2 &= \left( \frac{m_\text{e} v_\text{e}}{m_\text{p} v_\text{p}} \right)^2 = \frac{1}{2} m_\text{e} v_\text{e}^2 m_\text{e} = \frac{1}{2} m_\text{p} v_\text{p}^2 m_\text{p} = \frac{K m_\text{e}}{K m_\text{p}} \\
\frac{r_\text{e}}{r_\text{p}} &= \sqrt{\frac{m_\text{e}}{m_\text{p}}}
\end{align*}
\]
ELECTROMAGNETIC INDUCTION

1. What physical law describes how changing magnetic fields can cause electromotive forces? Include (a) an explanation, (b) an equation, and (c) a diagram.

\[ E = -\frac{d\Phi}{dt} = -\frac{d}{dt}\mathbf{B} \cdot d\mathbf{A} \]

Increasing the area \( \mathbf{S} \) of a loop causes \( \mathbf{E} \) and \( \mathbf{I} \) to induce circular current.

2/What is the direction of the induced current in each circular loop, due to the current shown in each diagram. Show your reasoning, including (i) direction of the original magnetic field, (ii) direction of the induced magnetic field in the loop, (iii) direction of the induced current.

3. Consider a coil of area \( A \) spinning in a magnetic field \( B \), as shown in the figure.
(a) Describe how this motion can result in an electromotive force (or voltage) around the coil.

The magnetic flux through the loop changes as the loop spins.

\[ \frac{d\Phi}{dt} = \frac{d}{dt}(B \cdot A) \]

(b) Justify each step of this equation with words:

Faraday's Law:

Flux \( \Phi \) = amount of field \( B \) through area \( A \) through loop.

(c) If the angle \( \theta \) between the magnetic field and the loop area changes due to the spinning frequency \( \omega \) of the loop as \( \theta = \omega t \), then find the simplest expression for the emf \( E \) in a coil with \( N \) loops.

\[ E = -N \frac{d}{dt}(B \cdot A) \cos \theta \]

\[ \frac{d}{dt}(B \cdot A) \cos \theta = -\omega B \sin \omega t \]

\[ E = -NBA \omega \sin \omega t \]

This is a generator: motion making electricity in the presence of a magnetic field.
IV. ASTROPHYSICS: Only Math-B students need do this part (20-30 minutes)

1. Describe how you can find each of these things about a star from observations. Use (i) words and (ii) a simple equation (or a diagram)

(a) Temperature \( T \), spectrum, color \( \rightarrow \) peak wavelength \( \lambda_{\text{peak}} = 3 \times 10^{-3} \) K

(b) Radiant Flux \( F = 4\pi r^2 \) you know from \( T \) which we know from spectra

(c) absolute magnitude spectrum + HR diagram

(d) apparent magnitude count photons or compare brightness to known star

(e) distance
\[ m - M = 5 \log \frac{d}{10} \text{ pc} \]

(f) luminosity compare the magnitude to that of stars of known \( L \)
\[ M - M_0 = \frac{5}{2} \log \frac{L}{L_\odot} \]

(g) radius \( \frac{R}{A} \rightarrow L = 4\pi R^2 \rightarrow R \) from \( L \) (magnitude) and \( T \) (spectrum)

2. Consider a model of a star consisting of a spherical blackbody with a surface temperature of 10,000 K and a radius of 3 \( R_\odot \). Let this model star be located at a distance of 200 pc from Earth. Determine the following for this star.

(a) Radius
\[ \text{Radius} \]

(b) Luminosity of the star
\[ \text{Luminosity} \]

(c) Radiant flux at the star's surface
\[ \text{Radiant flux} \]

(d) Radiant flux at Earth's surface (compare this with the solar constant)
\[ \text{Radiant flux} \]

(e) Peak wavelength
\[ \text{Peak wavelength} \]

(f) What kind of star would this be? an A or B star, bright white like Vega

(a) \[ \lambda_{\text{(A)}} = \frac{3 \times 10^{-3}}{T(k)} = \frac{3 \times 10^{-3}}{10^4} = 3 \times 10^{-7} \text{ m} \ (UW - 10,000 \text{ blue-white}) \]

(b) \[ F = \frac{C}{4\pi r^2} = 5.67 \times 10^{-8} \text{ W/m}^2 \text{K}^4 \times (10^4)^4 = 5.67 \times 10^{-8} \times 10^{16} = 5.67 \times 10^8 \text{ W/m}^2 \]

(c) \[ L = F \times \text{Area} = F \times 4\pi r^2 = 5.67 \times 10^8 \text{ W/m}^2 \times 4\pi \left(3 \times 10^8 \text{ m} \right)^2 \]
\[ L = 6 \times 12 \times 10^8 \times 8 \times 10^{18} \approx 50 \times 10^{36} \text{ W} = 5 \times 10^{38} \text{ W} \]

(d) \[ F_{\text{at earth}} = \frac{L}{4\pi (6 \times 10^{18} \text{ m})^2} = \frac{5 \times 10^{28} \text{ W}}{4\pi (6 \times 10^{18} \text{ m})^2} = \frac{5 \times 10^{28} \text{ W}}{12 \times 36 \times 10^{36} \text{ m}^2} \approx \frac{5 \times 10^{28} \text{ W}}{5 \times 10^{10}} \]
\[ F_{\text{at earth}} \approx 10^{-8-2} = 10^{-10} \text{ W/m}^2 \]