For this assignment will use the recursion features of Excel to evaluate at least 20 terms of the suggested sequences. Before you start with the questions place the values $1, 2, 3, \ldots 20$ in column A. Remember that when you change a recursive formula in one cell you must make sure to copy that formula down to all the other cells that use it.

Answer all the questions using complete sentences in a text box in Excel. Drop your solutions in the DropBox in the Emerging Order program space on MASU using the naming convention:

`Recursion_Lab_LastName_FirstName.xls`

1. Use Excel formulas to generate the following sequences
   (a) The triangular numbers;
   (b) The tetrahedral numbers (Hint: remember these are formed when you stack balls which are arranged in layers of triangles – ie the tetrahedral numbers are the sum of the triangular numbers.);
   (c) the Fibonacci Sequence.

2. A sequence is said to **converge** to a **limit** if the terms get closer and closer to a fixed value. For example the sequence 1, 1.5, 1.75, 1.825, 1.9375 ... appears to converge to the limit 2. If a sequence does not converge to a limit it is called a **divergent** sequence. A divergent sequence can diverge to infinity (*eg* 2,4,8,16,32 ....). It can oscillate between two or more values (*eg* 0,1,0,1,0,1 ...), in which case it is called a **periodic** sequence. Or it may seem to exhibit a apparently random pattern in which case it is **chaotic**.

   Generate each of the following sequences and classify them as above. If the sequence converges write down the limiting value. Note: Sometimes you will need to generate a large number of terms before you are certain about the convergence of a sequence.

   (a) $u_n = \dfrac{1}{u_{n-1}}$ and $u_1 = 2$
   (b) $u_n = 1 + \dfrac{1}{u_{n-1}}$ and $u_1 = 1$
   (c) $u_n = u_{n-1} + 2n - 1$ and $u_1 = 1$
   (d) $u_n = 3u_{n-1}(1 - u_{n-1})$ and $u_1 = 0.5$

3. **Emerging order in two dimensions:** One of themes of this programs is how basic elements following simple rules can build complex structure. The following example shows how a one in a sea of zeros can generate an amazing two dimensional pattern of numbers, which contains within in it many of the sequences we have been exploring.

   In a new spread sheet (say sheet 2) enter zeros in the first 15 columns of row 1 then put the number 1 in cell P1. (You may need to resize the columns so that it all fits on one screen). The rule we will use for generating the elements in the next row is a simple one. Add the value in the cell immediately above the one you wish to fill to the one in the next column to the right. For example in cell A2 you should write **=A1+B1**, and in B2 write **=B1+C1**.

Use the copying features in Excel to fill row 2 with the same formula. Then select the entire row 1 and copy down to row 16 to apply the same formula to all cells. What you should have generated is called Pascal's Triangle – a fascinating sequences of numbers, discovered in China, that we will keep coming back to. In a text box describe and state a recursive definition for the following sequences:

(a) the sequence in column N

(b) the sequence in column M

(c) the sequence formed by summing the values in each row (use the Excel sum function to generate these numbers)

(d) Find and highlight the numbers in the diagonal lines whose sum leads to the Fibonacci numbers (This one is not obvious but it is there).

(e) Make a separate copy of Pascals triangle and then highlight all the cells with odd numbers in red. The pattern you get is called the Sierpinski Gasket. This is a fractal pattern that we will revisit at the end of the quarter.