

1. Write down the equation of the following spirals and describe them as logarithmic or Archimedean

- (a) A spiral that starts with a radius of 10 cm and grows by 2 cm every revolution.

This is an Archimedean spiral because the radius increases by a constant amount each revolution. Therefore,

$$r = 10 + 2\frac{\theta}{360} = 10 + \frac{\theta}{180} .$$

- (b) A spiral that starts with a radius of 2 cm and grows by a factor of 3 every half turn.

This is a logarithmic spiral because it grows by a constant factor every 180 degrees. Therefore,

$$r = 2(3)^{\frac{\theta}{180}} .$$

- (c) A spiral that starts at 2 cm and grows by 50% every 90° .

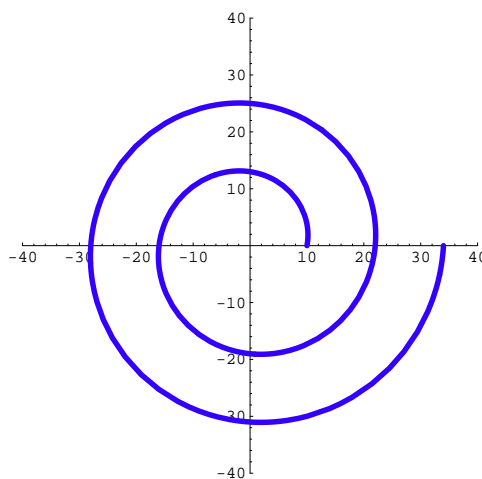
Since the spiral grows by 50% every 90° , the radius grows according to a geometric sequence 2, 3, 4.5, \dots which growth factor 1.5. Therefore,

$$r = 2(1.5)^{\frac{\theta}{90}} .$$

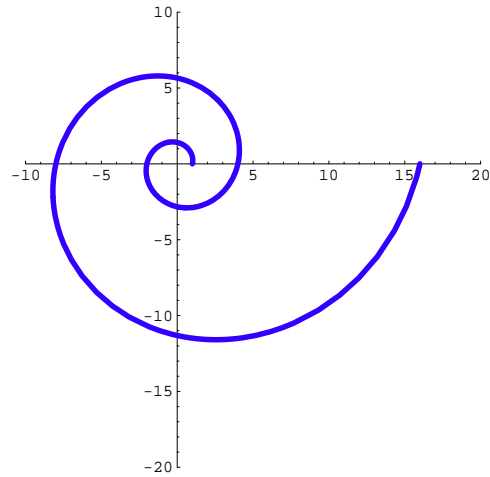
Note: If the radius grows by $x\%$ then the growth factor is $1 + \frac{x}{100}$.

2. The mathematical expressions for the spirals we have discussed are examples of polar functions. Such functions give the coordinates of a point on the plane by specifying a radius, r and angle θ , rather than the traditional cartesian coordinates x and y . Polar functions are plotted using a polar coordinates system and often reveal interesting graphs. Each member of your workgroup should choose one of the following functions to plot. Make a table of values for the radius r and the angle θ . Start with $\theta = 0^\circ$ and go up to at least 720° in increments of 30° . Plot the points on the polar graph paper provided and connect the points in sequential order. (Note: In some cases you will get a negative value for r . In such cases plot the point in the opposite direction to that indicated by the angle θ .)

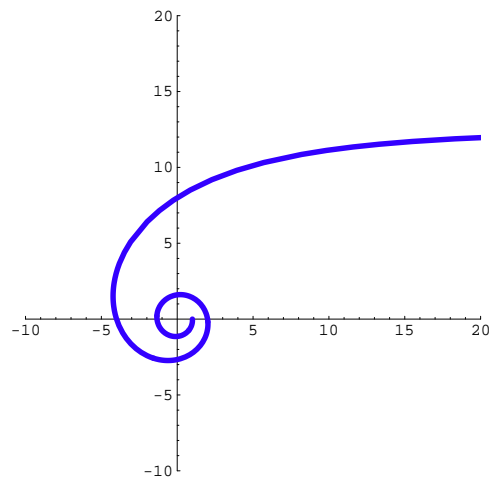
- (a) $r = 10 + \theta/30$. Archimedean Spiral



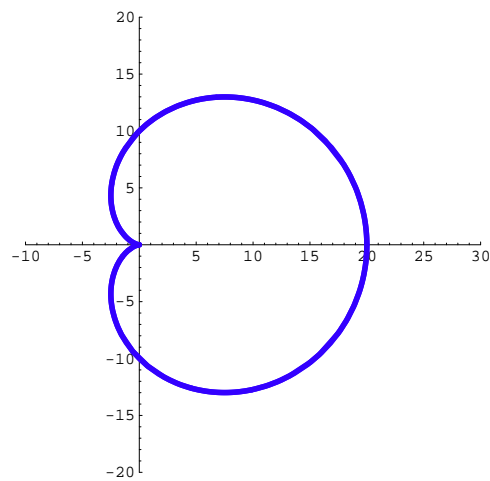
(b) $r = 2^{\theta/180}$. Logarithmic spiral



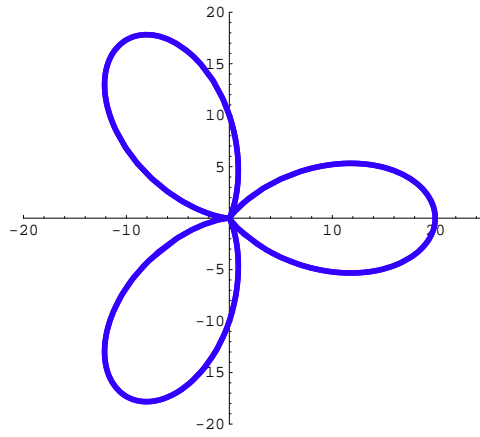
(c) $r = 720/\theta$ and $\theta \neq 0$. Hyperbolic spiral



(d) $r = 10 + 10 \cos \theta$. This curve is called a cardioid. What does it remind you of?



(e) $r = 10 + 10 \cos 3\theta$. This curve is called a trifolium. What does it remind you of?



Sketch each function plotted by your group members on the back of your graph paper. Describe them with appropriate mathematical language.