Introduction:

Branching patterns are a common and diverse phenomena in nature - occurring in trees, leaf venation, streams, lightening bolts, lungs, arteries and many other places. The geometric nature of these branching patterns have broad similarities. These similarities arise due to the common reason behind the branching phenomena - the distribution of energy to or from a large area in the most efficient way. We will complete two activities relating to branching. One is about energy flow, the other about quantification of branching.

Activity 1: Patterns of Energy Flow

In this activity we will demonstrate how the requirement for efficient distribution of energy dictates the nature of the branching. There are two competing principles. First, we would like to connect all points in the area to the source of energy by the most direct route possible. Second, we want to minimize the total length of the branching material (eg the veins).

1. On the top half of one piece of triangular grid paper mark, with a pencil, a hexagon of dots which is four dots on each side (locate the dots on the intersections of the grid lines). This hexagon represents an area the requires a source of energy or nutrients. Mark the central dots with a pen. The central dot is the source of nutrients for the rest of the dots. Starting at the center dot, create a single path that does not cross itself and passes once through every dot on the grid. You can think of this path as a feeding tube for the points on the grid. The type of path you have created is called a meander. Compare your path with those of your group. There will likely be a few different paths which satisfy this simple rule. Now calculate the following quantities for your path:

   (a) the total path length.

   (b) the average distance of each point on the grid from the center – measuring along the path connecting it to the center. To do this carefully you should make a list of distances for each point below and then find the average.

2. Now try a different strategy for feeding the points. Create a second hexagon of dots in the middle of your paper. This time connect each dot directly with the center. You do not need to follow the lines on the grid. This kind of pattern is called an explosion, or star burst. Find

(a) the total path length.

(b) the average distance of each point on the grid from the center – measuring along the path connecting it to the center.

Note: For both these calculations you will have considerably less work if you use the symmetry of your grid to help you calculate the distances. Compare your answers in this question with those in question 1. Can you say which is more efficient?

3. Now try to find a compromise between the meander and star burst patterns. On the bottom of the page draw out a new hexagon of dots as before. Create a system of paths from the center which include some kind of branching. Make sure there are no loops – that is, make sure that for every point there is only one path back to the center. (note: a path does not need to follow along the lines of the grid.). Compare your paths with those of your group members.

   (a) the total path length.

   (b) the average distance of each point on the grid from the center – measuring along the path connecting it to the center.

   How does your branching pattern do at minimizing the desired quantities?

Activity 2: Quantitative Analysis of Branching (Complete this in pairs).

1. Mark off a rectangular array of dots on your grid which is 10 dots wide by 20 dots high. For the sake of this activity think of this area as a drainage field for a river. Water at each point will drain down left or right to the next point. For each dot in your grid toss a coin. For heads draw a line in pencil going down to the left to the next point. For tails draw a line to the right. After you have completed this for all points in your grid you will have a collection of lines representing the flow of water down a gradient. (The observant among you may recognize this pattern as similar to one formed by the receding wave at the beach). Some of these lines will have naturally joined into branched networks. Find the longest such network and colour it.

2. For this random branching network complete the following analysis. First label the branches as first order, second order or third order. First order branches are those that originate from a point that has no other lines. Second order branches are those that originate when two first order branches join. A third order branch is one which originates when two second order branches and so on. Note when a first order branch joins a second order branch the second order branch does not changes its order. It may be helpful (and even aesthetically pleasing) to shade second order branches thicker than first order branches and so on. To complete the analysis find the ratio of the number of first order to second order branches. Then find the ratio of second order to third order branches and so on. Typically these ratios will range between 3 and 5 for streams. When branching networks are large, this ratios are remarkably constant and quite consistent over a large class of branching phenomena.

What are the quantitative similarities and differences you observe between the random stream branch network in Activity 2 and the constructed branching pattern in Activity 1.