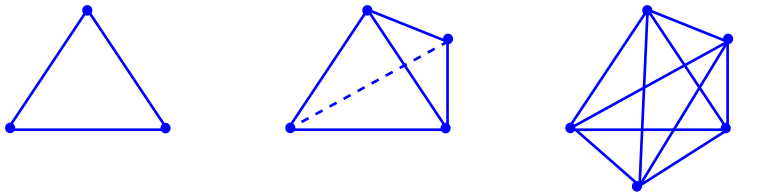
1. It is possible to generate another 4-dimensional shape in a similar way to the construction of the hypercube. Starting with a point add another point above it and connect the two points with a line segment. Now add another point to the right of the line segment and connect the previous two points to this point. This encloses a triangular area. The the triangle lies in a plane. If another point is added above the plane (in the third dimension) and all the vertices of the triangle are connected to the new point a tetrahedron is constructed. Adding another point in the fourth dimension and connecting each vertex of the tetrahedron to the new point defines a *hypertetrahedron* or a *pentahedroid*. Draw one diagram for each of the figures described above.

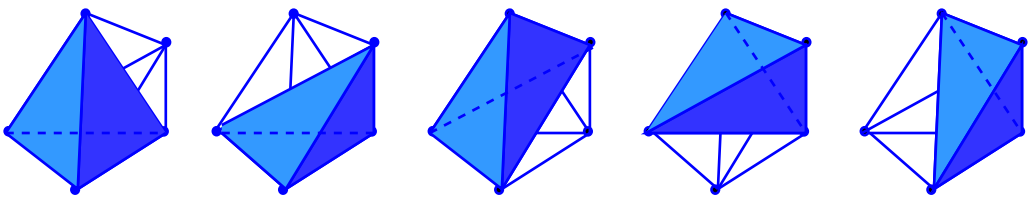


triangle tetrahedron hypertetrahedron

2. Use your diagrams in the previous question to help you fill in the table below (where solid refers to the number of solid regions enclosed by the figure and hyperregion refers to the number of 4-dimensional regions enclosed)

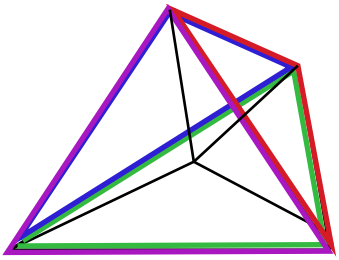
Dimension	Figure	vertex (v)	edge (e)	face (f)	solid (s)	hyperregion (h)
1	segment	2	1	0	0	0
2	triangle	3	3	1	0	0
3	tetrahedron	4	6	4	1	0
4	hypertetrahedron	5	10	10	5	1

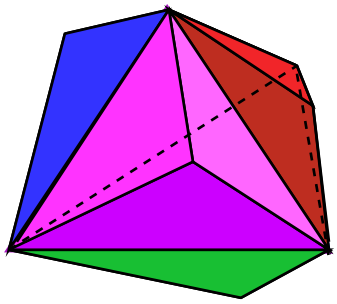
3. Shade each of the solid regions of the hypertetrahedron using a separate diagram of the hypertetrahedron for each region you shade.



4. Calculate the Euler characteristic $\chi = v - e + f - s + h$ for each of the figures in table in question two. What do you observe?
   In each case the Euler characteristic is 1.

5. Sketch an unfolding of the hypertetrahedron. Use shading to illustrate that the unfolding is a three dimensional figure.
   To see this put the fifth point of the hyper tetrahedron (the one in hyperspace) in the center of the tetrahedron. Then unfold along the lines connecting to this point. One tetrahedron will "pop out" of hyperspace attached to each face of the central tetrahedron.





6. Draw slices of a tetrahedron as it passes through a plane vertex first. Then draw slices of the hypertetrahedron as it passes through three-space vertex first.
   The slice of the tetrahedron will be equilateral triangles, and the slices of the hypertetrahedron will be tetrahedra.