HEINRICH HERTZ AND WITTGENSTEIN'S TRACTATUS*

1. WITTGENSTEIN'S GESTURES TOWARD Hertz

In one of the manuscript arrangements that Wittgenstein made near the end of his life of the material which eventually became the Philosophical Investigations, he chose the following motto from Heinrich Hertz's Principles of Mechanics:

When these painful contradictions are removed, the question of the nature [of force] will not have been answered, but our minds, no longer vexed, will cease to ask illegitimate questions.

In this passage Hertz is summarizing his central philosophical motivation for the startlingly original theoretical work of the Principles. By presenting in detail three alternative formulations of mechanics, and examining the factors which are involved in the actual choices scientists make between such alternative theories, Hertz thinks he can show that there are some questions which seem on their face to make sense—which seem to cry out for answers—but which can be shown to be unanswerable, and, in some sense, not meaningful questions at all. The connection with Wittgenstein's project in the Investigations is obvious. What Hertz had tried to do for our understanding of the language of physics, Wittgenstein was doing, in the Investigations, for our understanding of language in general.

There are a number of other places, going back to his earliest writings, where Wittgenstein explicitly refers to Hertz's work. Hertz's name appears twice in the Tractatus, twice in the Blue Book and several times in the Notebooks. There is a frequently cited passage published in Culture and Value where Wittgenstein gave a list of thinkers who had influenced him. The list runs: Boltzmann, Hertz, Schopenhauer, Frege, Russell, Kraus, Weininger, Spengler and Sraffa. The enduring nature of Hertz's influence has been characterized as follows:

The admiration that Wittgenstein conceived for Hertz in his youth was something he never lost. Later in life we find him entering reservations about almost everyone else—even about Frege—but right up to the end of his years he continued to quote Hertz with approval and agreement.

The fact of Hertz's persistent influence on Wittgenstein's thinking has been frequently noted, and the content of that influence has been indicated in broad strokes by a number of commentators and biographers such as Monk, McGuinnes, Pears, Baker and Hacker, and
Typical of these characterizations is McGuinness's statement that, while the young Wittgenstein was not much influenced by the writings of Ernst Mach, he "learnt much more from Hertz and Boltzmann: here he found the idea that science was a picture or model created by the mind, often with the utmost daring and freedom . . . 'The whole task of philosophy,' he was fond of quoting from Hertz, 'is to give such a form to our expressions that certain disquietudes (or problems) vanish.'" These writers consistently note that Hertz, like Wittgenstein, had some kind of a "picture theory" and that Wittgenstein approved of Hertz's sentiment, expressed in the passage quoted above, about the pseudo-problems caused by asking illegitimate questions. However I believe that there are important details of Hertz's influence that have not been discussed and which not only reveal the depth and complexity of that influence, but also contribute significantly to solving some vexing problems in the interpretation of Wittgenstein's writing.

In this paper I focus on Hertz's influence on the Tractatus since I believe that that influence is even less well understood than the influence on the later writings. This will require a careful examination of Hertz's philosophy of science, but the effort will be rewarded with important insights into some central issues in the Tractatus.

2. HISTORICAL SETTING

During the years when Wittgenstein was studying for a career in engineering (1903-1908) the world of theoretical physics was undergoing dramatic changes. Not only were new discoveries and technical advances being made, but deeply philosophical issues about the nature of scientific knowledge were being widely debated. In a reaction against what he saw as fruitless metaphysical claims of scientists under the influence of a widespread Hegelianism, Ernst Mach had, as early as 1872, proposed a thoroughgoing phenomenalism as a way to guarantee the empirical respectability of science. He claimed that the entire domain of physical science was contained in human sensation. Science can describe this sensory reality by using symbols which efficiently express recurrent types of sensations, but science cannot explain this reality without positing metaphysical entities beyond the reach of actual perception. Mach urged that such metaphysical posits be aggressively rooted out wherever they were found.

Helmholtz's and Hertz represent a neo-Kantian reaction to Mach, an attempt to reassert the importance of the free achievement of the human mind in the structuring of scientific explanation. Helmholtz had taught that "a complete and adequate understanding of nature
could be reached only through an explanation of its mechanism." This might involve positing unobserved entities such as concealed masses, atoms and forces of attraction and repulsion, entities which were anathema to Mach. In 1889 Helmholtz expressed his ambitious program as follows:

The work of science will have been completed only when phenomena have been traced back to the simple forces, and when it can be shown also that the given account is the only possible one admitted by the phenomena. Then this would have been shown to be the necessary way of interpreting nature, and it would be the one to which objective truth should be ascribed.

It fell, however to Hertz, who was Helmholtz's student, to formulate the most insightful and plausible reply to Mach's phenomenalism. Published in 1894, the year of his untimely death, Hertz's *Principles of Mechanics Presented in a New Form* begins with a long and philosophically innovative introduction. He breaks with his teacher by suggesting that the aim of science need not be tied to the hope of finding a single theoretical formulation which will give a complete explanation of nature. Rather, science should examine a variety of possible formulations and be willing to adopt various alternatives which may be found to be better suited to various divergent purposes. In this he is less dogmatic than Helmholtz, and his scientific pragmatism has much in common with Mach's. Furthermore, as a highly successful practitioner of the experimental method, Hertz's credentials as a practical empiricist were unassailable. Even on the theoretical level he was in agreement with Mach that "every idea employed by physical theory would eventually have to be confirmed by concrete observation. However, he was convinced that not every single element of a theory is susceptible to or in need of such verification." An element of a theory might be "verified" simply because it was part of a theory which, as a whole, made correct predictions of observable phenomena. This "holism" about theory confirmation is what signals a basic disagreement with Mach. "For Hertz the fundamental concepts of theoretical physics were patterns of possible experience, whereas for Mach they were copies of actual experiences."

Poincaré continued and developed this line of thought, broadening it beyond mechanics to include scientific concepts and hypotheses in general and mathematical hypotheses in particular. Just as for Hertz different and mutually incompatible representations of mechanics might prove useful for different purposes, Poincaré argued that "no Geometry is 'truer' than another, though one may be more suited to the purpose of experience, or, in other words, prove a more useful instrument for a systematic description of its given facts."
Hertz's insight that scientific theories are confirmed as wholes, not piecemeal, is most familiar today in the trenchant formulation given by Duhem in his 1906 La Théorie physique. For whatever reason, Poincaré's and Duhem's reputations as pioneers in the philosophy of science have tended to overshadow the importance and originality of Hertz's achievement. This would not, however, have been the case in the Austria and Germany of Wittgenstein's youth.

Hertz's genius as both a practicing scientist and a philosopher of science is aptly commemorated by the fact that his name appears in computer specifications and on radio dials all over the world (in the guise of "kilohertz" and "megahertz") as well as in the pages of the Tractatus. His pioneering work with electromagnetic waves led to this first honor, his views about the nature of scientific representations led to the second. His other contributions to theoretical physics, while of great importance to the development of that discipline, have not been so dramatically commemorated. Perhaps the most important of these, in its practical significance, was his development of the concept of "configuration space," a high dimensional conceptual space in which the state of a complex physical system can be represented by a single point. The "phase space" of statistical thermodynamics is a variation of this idea as is the "Hilbert space" of quantum theory. Most importantly for our concerns, Wittgenstein's notion of "logical space" is, I will argue, a generalization of Hertz's concept of configuration space to the case of linguistic representation. This realization, I believe, sheds important light on the related Tractarian notions of "representational form," "logical form," and the "projective" aspects of propositions.

The fact that Hertz expressed his theory of mechanics in terms of configuration space has led to his being credited with a prophetic anticipation of Einstein's theory of relativity. Cornelius Lanczos, in a historical survey of the most important figures in 19th and 20th theoretical physics⁹ says:

The theory of relativity, starting from entirely different considerations, and developing along different lines, provided an impressive example of the forceless mechanics of Hertz. [In relativity theory] planetary motion around the sun was explained as caused by pure inertia, without any acting force. The planets trace out shortest lines in a Riemannian space, just as Hertz imagined for mechanical systems which are free from potential energy. The only difference is that in Hertz's system the Riemannian curvature of the configuration space is caused by kinematical conditions imposed on the hidden motions of the system, while in Einstein's theory the Riemannian structure of physical space-time manifold is an inherent property of the geometry of the world.
The details of this characterization will become clearer as we examine the details of Hertz's project. What I hope is apparent at this point, however, is the relevance of Hertz's theoretical insights to some of the central problems facing early 20th century science.

In view of Hertz's impressive accomplishments, and notwithstanding his current obscurity, we should not be at all surprised that he could exercise a strong influence on a philosophically minded student of physics in Austria and Berlin in the first decade of this century.

3. HERTZ'S PROJECT

Hertz begins the introduction to the *Principles* by stating his view of the most general features of human knowledge. The central task of knowledge is to guide our practical choices by allowing us to anticipate future events. We need to be able to draw inferences about the future based on past experiences. In order to do this "we form for ourselves inner images or symbols of external objects"\(^1\) These "pictures ... are our conceptions (Vorstellungen) of things."\(^11\) He takes it as a common characteristic of human experience that we have such images, and that these images have one characteristic mode of correspondence (Übereinstimmung\(^12\)) with external nature. This correspondence consists in the fact that "the cognitively necessary consequences of the images be in accord with the images of the physically necessary consequences of the things pictured."\(^13\) Hertz characterizes this correspondence as a requirement (Forderung) which our images must meet in order to fulfill their purpose (Zweck), allowing us to anticipate the course of events in nature (in other places he refers to this as "the fundamental requirement" to distinguish it from other subsidiary requirements which, as we shall see, may be imposed upon our images).

This crucial relation of correspondence, the representational relation which was later at the center of the Tractarian "picture theory," is here left without further elaboration beyond the statement that experience (of the pragmatic success of both common sense and science) tells us that such a correspondence does indeed exist.

But Hertz makes another strong claim about the relation between our ideas and their objects. He says that this pragmatic inferential correspondence is the *only* relation we can expect between the two. He suggests that there are features of things that are *not* reflected in our ideas, and which thus escape the power of our representations. In other words, it is the logical (inferential) relations between mental images, and the *use* of those images as tools for coping with the world, that mark the essential features of human knowledge.
Having sketched this provocative picture of knowledge in general, Hertz turns his attention to some of the details that are visible in the conduct of one field of knowledge, physical science. In ordinary experience we form, more or less automatically, ideas or images of objects. These presentations (Vorstellungen) are organized and refined by science into representations (Darstellungen) of nature. These scientific representations Hertz also call pictures (Bilder, sometimes translated 'images', or 'models'). Since the properties of our images of nature are always underdetermined by the properties of the things in nature which they represent, alternative images are always possible. Hertz's project is to consider alternative theories of mechanics. Each theory, or system, is a picture, or representation of the world. In order to pass judgement on the adequacy of a particular system we need to consider additional requirements (Forderungen) that we make on scientific representations. Hertz lists three. We require that any formulation of mechanics be a) logically permissible (zulässig), b) empirically correct (richtig) and c) suitable to our purposes (zweckmäßig). It is important to understand what he meant by each of these terms.

Hertz explains that logical permisibility means that the theory may not "implicitly contradict the laws of our thought." However this requires more that just that the theory be formally consistent when axiomatized in a canonical notation. There are also "logical" constraints on how the formal notation is interpreted and applied. This is clear from the doubts which Hertz raises about the permissibility of Newtonian mechanics. He is not worried about finding formal contradictions in the formulae of Newton's system; or rather, he asks us to examine carefully the sense which we attach to the word "force" in its various occurrences in Newton's Laws of Motion. He claims to detect a slight shift in meaning between two of its occurrences, and this shift, if it is really there, is enough to cause the theory to fall short of fully meeting this requirement. The logical flaw he is worried about here is not formal inconsistency but rather the fallacy of equivocation.

The second requirement, correctness, is just a specific application of the "fundamental requirement" on all our ideas and conceptions (Vorstellungen) to the special case of scientific representations (Darstellungen): the consequences of the pictures must be pictures of the consequences. Any theory that does not meet this requirement is a non-starter and subject to immediate rejection. The complication with this requirement is that most theories imply many consequences that have not, and perhaps cannot be observed. These theories always stand at risk of being proved wrong by the next observation. This is a situation that Hertz is content to accept as an inevitable feature of all our scientific theories.
Zweckmäßigkeit, the third requirement, deserves special attention. The word is usually translated "appropriateness" or "suitableness." The roots of the word are Zweck, a purpose, aim or goal and Maß, a measure or proportion. Something is zweckmäßig if it "measures up to a purpose." "Appropriateness" is an adequate translation if we keep in mind that it does not indicate an absolute standard but is always applied relative to a particular human purpose. There are two distinct purposes that Hertz mentions for which a scientist might formulate a picture of mechanics. One is "practical applications to meet the needs of mankind." This, he emphatically states, is not his purpose in writing the Principles. His purpose, rather, is to create a picture of maximum clarity and simplicity and in so doing show something important about the nature of physical theory in general. He explains the goals of clarity and simplicity by describing in general terms the formal structure of a physical theory and its relation to nature. The "principles" of a theory are:

a) its primitive concepts,

b) statements of the relations between those concepts (i.e., the laws of the theory),

c) explanations of how those basic concepts are related to human experience.

The theory proper contains the principles and all their deductive consequences, interpreted as statements about possible experience. Any given theory, while being totally correct, can fail to match nature in two ways: It can say too little or it can say too much. If it fails to say anything about certain kinds of experiences with which we are familiar, it is, to that extent, unclear (Hertz's term, 'undeutlich' might also be translated 'vague' or 'uninformative'). If its expressions contain relations which do not have experiential counterparts, it says too much and to that extent lacks simplicity (Einfachheit). We can't expect any theory to explain everything, and every theory, since it is a symbolic representation, will contain as part of its "mode of picturing" (Abbildungsweise) some features that do not correspond to anything in the nature of what it represents. Every theory strikes its own particular balance between clarity and simplicity while pursuing other (usually pragmatic) purposes.

By designing a formulation which purposely ignores questions of practical applicability in favor of maximizing clarity and simplicity Hertz thinks he can provide an important insight into the nature of scientific representation, an insight that will put to rest certain nagging "foundational questions" such as "What is Force?" It is in this context that he makes the statement about "ceasing to be troubled by illegitimate questions," the statement which impressed Wittgenstein so strongly.

By making explicit some of the requirements we make on scientific representations, Hertz has put us in a position to ask where these requirements come from. He follows Kant in
seeing the logical requirement as arising from the nature of the human mind (specifically from the forms of our intuitions and judgements). The standard of correctness requires conformity to actual or possible human experiences and implies that scientific representations do not have any meaning beyond those limits, again a Kantian criterion. Finally the requirement that a picture be suitable to our purposes leaves open the possibility that we may choose one representation for one purpose and another for another. The means by which we meet these purposes, the representational forms, are the product of the action of the human mind, constrained in certain ways, to be sure, but with a broad field of freedom within which to invent new forms for new expressive purposes.

4. ONE PICTURE OF MECHANICS: NEWTON'S THEORY

As examples of this creative activity Hertz then describes three representations of mechanics. The first is classical or Newtonian mechanics, with its four primitive concepts: space, time, mass and force. The relations among these four are expressed in various definition and in the fundamental law: \( F=ma \). The picture is filled out by the other laws of motion and d'Alembert's principle. In spite of the monumental success of this theory (and without the benefit of the radical challenges posed to it by quantum mechanics and relativity theory) Hertz complains that the classical theory fails to fully satisfy any of the three requirements we make of scientific representations.

First, Hertz objects to the conceptual ambiguity about force, which was noted above, and which we will now examine in detail. In the second law force appears as a preexisting cause of motion, but in the third law it is a result of motion. Hertz detects a certain embarrassment in the traditional presentation of the foundational principles of Newtonian theory, a certain impatience to get on to the applications of the theory where the predictive results speak for themselves. He attributes this foundational insecurity to the historical process of the theory's development, focused as it was from the beginning on serving certain fairly narrow explanatory purposes. The result was that certain features of the facts to be explained were allowed to influence the formal structure of the theory.

For example, in calculating planetary orbits, the planets could be treated, for convenience, as if all their mass were concentrated at a single point and the total gravitational and inertial forces were acting on that point. But this is obviously an idealization. What the Force Law says is that the motion of each quantity of matter is determined by the forces acting upon it alone. Since the various parts of a planet are different distances from the sun, the solar gravitational force on different particles is different and the paths of the various particles
should gradually drift apart. In the case of planets, the fact that this drifting apart does not occur can be explained by the fact that in addition to solar gravitation, there are other gravitational forces which the parts of the planet exert on each other. The forces holding the planet together are large relative to those trying to tear it apart. But now consider the motion of smaller objects, whose internal gravitation is negligible. The spinning of a top can develop tremendous inertial forces, which the Force Law says act independently on each particle of its matter. The parts of the top are all moving in different directions at different speeds. Calculating the motion of independent particles, based only on the gravitational and inertial forces acting on them leads to the result that the top should fly apart. But we do not observe this result. If the Force Law is true, there must be other forces involved to account for the rigid connections observed. We have not given a complete account of the motion of the top until we have added in these "cohesive forces". If the top is magnetic and rotating in an electrical field there will be other unaccountable discrepancies in its motion and yet further forces will have to be posited. In Hertz's day the number of forces seemed to be increasing steadily. Each new force will be introduced as a "something, I know not what," a numerical quantity which supplements the force in \( F=ma \) to exactly the extent required to account for the observed motions and rigid connections. The fact that this patching up could be carried out in a way that continued to yield reasonably good predictions did not lessen the feeling that there might be a simpler way to represent the phenomena in question. "Such groping and pseudo-certainty" Hertz says, "is unacceptable" in a mature science, and leads to disturbing questions like "Why do the various acting forces and inertial forces behave in such different ways?" and "What is Force really?" Not only were scientists compelled to posit new kinds of forces, but the theory also allowed certain motions which violate the conservation of energy, and this required assuming new restrictions on the range of the law. Furthermore, many of the explanations, although predictively accurate, included deductive steps which referred to entities which appeared to be \textit{ad-hoc} computational conveniences, which were brought into play for making predictions in a particular field of physics, and were then ignored when working in another. Most unsettling to Hertz was the fact that these forces were supposed to exist in states of equilibrium that made it possible to conveniently ignore them when describing the motions of the macroscopic objects for which the theory had originally been contrived. He asks us to consider an iron bar resting on a table. Newtonian mechanics describes the situation as follows:

Through the force of gravitation every atom of the iron is attracted by every other atom in the universe. But every atom of the iron is magnetic, and is thus connected by fresh
forces with every other magnetic atom. . . . Again, bodies in the universe contain
electricity in motion, and this exerts further complicated forces. . . and in addition to
these, various kinds of molecular forces. . . . If only a part of these forces were effective,
this part would suffice to tear the iron to pieces. But in fact, all the forces are so adjusted
. . . that the effect of the whole lot is zero; no motion takes place. . . . Now if we place
these conceptions before unprejudiced persons, who will believe us? Whom shall we
convince that we are speaking of actual things, not images of a riotous imagination?

Might there be simpler ways of describing the motion of familiar objects? At least there are,
he suggests, some plausible candidates.

5. A SECOND PICTURE: ENERGETICS
Investigations of the motion produced by heat and other transformations of one kind of
energy into another led to the appreciation of the importance of the principle of conservation
of energy and eventually to the development of what Hertz calls the second picture of
mechanics.

This line of thought, sometimes called "energetics", has as its four primitive notions:
space, time, mass and energy (force being introduced by definition in terms of the others).
Although at the time Hertz was writing there was no complete formulation of mechanics
along these lines, there were specialized fields in which "complete chains of reasoning were
carried out entirely in this mode of thought."

One of the central insights of this approach is that if we consider a moving body only in
terms of its mass, location, and the work being done in causing its motion, we can safely
ignore all the rigid connections it it may contain. Since no work is required to maintain
those connections, they need not be represented. Further, by thinking of motion as a
transition from the body's being in one location at a given time to its being in another at a
later time, a complete description will only require explaining why it took the path it did and
what work was expended in the transition. This allows for a tremendous simplification of the
representational apparatus. On this picture the state of a body at a given time is determined
by its mass and the general energy properties of the system of which it is a part.

This "Second Picture" Hertz tells us, had also produced important predictive and
explanatory results (much of his own work laboratory had been done within this framework).
But again there are problems, both conceptual and empirical. The first complication is the
need to divide energy into two distinct types, kinetic and potential. The basic law for such a
system states the relation between the kinetic and potential energy that accompanies motion.
This relation is given by a minimum principle such as Euler's Principle of Least Action or
Hamilton's Principle. There are, accordingly, several more or less equivalent ways to develop
a consistent system and Hertz chooses for examination a version based on Hamilton's Principle.

Such a system is a clear improvement over Newtonian physics in terms of its appropriateness to certain experimental and explanatory purposes. It displays more of the constraints which are actually observed in natural motions. Since Hamilton's principle can only be deduced from Newton's laws by making the additional assumption that all forces are conservative, the former is a more informative principle.\textsuperscript{25}

There were, however, other areas of mechanics to which this picture was not well suited. There are certain kinds of observed motions (e.g. the rolling of one body on the surface of another) which violate Hamilton's principle. These problems could only be fixed by positing unknown forces (usually friction of some kind) which brought with them the problems endemic to the first image, or else by limiting the range of applicability of the theory, and thus compromising its overall informativeness.

There was, in addition, a deep problem with the "logical permissibility" of this kind of theory, a question about the significance of the word "energy." The troubling duality in the concept of force between acting forces and inertial forces is echoed here in the contrast between kinetic and potential energy. Energy is treated as a kind of substance in so far as it is associated at a given time with a definite spatial locations and retains its quantity through changes of location.\textsuperscript{26} But kinetic and potential energy behave very differently and the theory only works by positing transformation of one into the other. Attributions of potential energy to bodies violate many intuitions we have about how substances can behave. For example, actual calculations often involve attributing negative quantities of potential energy to bodies, and the potential energy attributed to a given body is thought of as being determined by its external relations to other bodies, no matter how far away. Hertz considers the possibility that the universe might be infinite, in which case every body would possess an infinite amount of potential energy (conveniently balanced in all directions and thus having the observed small net value).

Finally Hertz is worried by the feeling that Hamilton's principle, as formulated in his day,\textsuperscript{27} was just too complex to be taken seriously as the fundamental law governing all natural motion. He anticipates that he will be charged with making "metaphysical" demands in requiring simplicity of our physical theories, but he says in reply "a doubt that makes an impression on our mind cannot be removed by calling it metaphysical. Every thoughtful mind as such has needs which scientific men are accustomed to denote as metaphysical."\textsuperscript{28} But in demanding simple theories we are not judging \emph{a priori} the level of complexity of the
universe, but rather saying something about the kinds of representations of that complexity we need in order to feel satisfied with our understanding. "Our requirement of simplicity does not apply to nature, but to the images thereof which we fashion."29

6. THE THIRD PICTURE: HERTZIAN MECHANICS
When he turns to the presentation of his own system, Hertz explains why we need to construct simple, logically consistent theoretical structures in order to understand the complexity of nature. Our experience of the motion of bodies around us is not obviously rule conforming (gesetzmäßig) and a pure description of that experience would be too chaotic to count as understanding. Wittgenstein refers to this passage at 6.361: "In the terminology of Hertz we might say that only rule-conforming (gesetzmäßige) connections are thinkable." In order to obtain a rule conforming world picture (gesetzmäßiges Weltbild) we need to posit something in addition to the visible motions, an unseen confederate, as Hertz says. But what to posit is a choice the scientist makes. The first two theories he has considered fill out the picture with force and energy respectively. But these turn out to be entities of a peculiar kind which are difficult to rigorously define. The alternative Hertz chooses is to fill out the picture using more of what is already most familiar, namely mass, time and space, but to allow some of the mass-in-motion to be undetectable. (In doing this he was at least no worse off than the Newtonian picture of his day, which had to admit undetectable forces of attraction and repulsion between particles; nor is he any worse off than Energetics, which, for example, explained the expansion of hot gases by undetectable increases in the kinetic energy of individual molecules.)

Accordingly, where the other two theories each had four basic concepts, his own third picture uses only three: space, time and mass. All these he can define clearly, and relate simply to common experience. None of these three is directly experienced in itself, but determinate quantities of each are experienced and measurable: Time by a good chronometer,30 space by an accurate ruler and mass by comparison with a standard object on an accurate balance. He then states one simple law which will account for all natural motions: *Every free system persists in a state of rest or uniform movement in a straightest path.* This law combines the significance of Newton's law of inertia and Gauss's principle of least constraint into a single statement. It is able to do this because of the particular mathematical structure within which the motions are described. The "straightest paths" referred to in the law are not straight lines in three dimensional Euclidian space but rather Riemannian geodesics in a multidimensional abstract construction: the 'configuration space'
mentioned above. By combining the mathematical techniques of Riemann's non-Euclidian geometry of curved surfaces and the geometry of multidimensional spaces Hertz was able to reduce the geometrical representation of complex systems of moving bodies to the geometry of a single point moving along a path in a formally structured abstract space.

Although this technique was soon to become standard procedure in physics, Hertz was very tentative and apologetic about introducing this "new, unusual and comprehensive mode of expression," and accordingly gives arguments for its advantages. The first is the simplicity of the representation; it allows the state of a complex system to be represented by a single point and motions of that system by the motion of a single point. Second, it allows the formulation of a fundamental law that contains the essential predictive information from both Newtonian mechanics and Energetics without referring to extraneous entities such as force or energy. Finally it makes clear the connection between, on the one hand, Hamilton's "characteristic functions," which had appeared to be applicable to only a restricted kind of motion, and, on the other, the universal laws of motion. In configuration space the characteristic functions are revealed to be special applications of a general geometrical technique with an a priori structure independent of its application to mechanical concepts. It was only because the idea of characteristic functions was formulated in a way that confused their empirical content with their formal structure that their physical significance seemed mysterious. A clear formulation of mechanics, such as Hertz proposes, "avoids the unnatural admixture of supra-sensible abstractions with a branch of physics."

This third advantage is significant because it once again illustrates Hertz's general theoretical concern with separating the formal structure from the content of scientific representations.

In carefully distinguishing between the form of scientific representations and their content, Hertz thinks he puts us, as consumers of scientific explanation, in a better position to choose the representation which best meets our present requirements. This possibility of making free choices among alternative systems of describing the world, and the connection between this possibility and the form/content distinction, is aptly illustrated in the Tractatus in a passage which is central to any understanding of Hertz's influence on that work. At 6.341 Wittgenstein discusses representational forms used by theories of mechanics. He introduces a suggestive metaphor, which I will call the "net analogy."

Newtonian mechanics, for example, imposes a unified form on the description of the world. Let us imagine a white surface with irregular black spots on it. We then say that whatever kind of picture these make, I can always approximate as closely as I like
(beliebig nahe) to the description of it by covering the surface with a sufficiently fine square mesh, and then saying of every square whether it is black or white. . . . The form is as I like (beliebig), since I could have achieved the same result by using a net with triangular or hexagonal mesh. Possibly the use of a triangular mesh would have made the description simpler: that is to say, it might have described the surface more accurately with a coarse triangular mesh than with a fine square mesh (or conversely). The different nets correspond to different systems for describing the world. Mechanics determines a form of description of the world by saying that all propositions used in the description of the world must be obtained in a given way from a given set of propositions—the Axioms of mechanics.

A crucial implication of this passage is that once we have adopted the practice of making factual statements about features of our experience, we are thereby committed to the use of some form or other (in the special sense of form with which both Hertz and Wittgenstein are concerned) in pursuing that practice. However exactly which form we use is a matter of choice and may be influenced by other projects and interests.

Consider what this might mean in actual practice. We might have two description of the orbits of the moons of Jupiter, one stated in the Newtonian idiom, the other in the Hertzian. Because the latter system has only three primitive concepts, its description of the motion of one of the moons can be stated in a way that refers to only three kinds of entities whereas the Newtonian formulation will refer to four. If both representations are correct, those descriptions will be made true by a single fact or set of facts, namely the actual motion of the moon. And yet they will be making statements about different entities. We see here a kind of ontological relativity which did not seem to bother Hertz at all. Since direct experience of force is not part of the data to be explained, it cannot count against the correctness of a theory that it fails to refer to forces. Various theories should get on with the job of giving explanations of the phenomenon that we care about without getting bogged down by questions about the "ultimate nature" of the entities to which the theories referred. If one is tempted to ask "What is Force?" being shown that all phenomena which can be described using force can just as accurately be described without it should lead us to the realization that force is a characteristic of a mode of representation, not of what is being represented.

This put Hertz in opposition both to any dogmatic rationalist view of science, which might claim to speak about the ultimate components of reality, and also to Machian phenomenalism which claimed to build a positive science based purely on sensation. As if to emphasize the fundamental difference between his view of science and Mach's, Hertz divided the actual presentation of his system in the body of the Principles of Mechanics into two parts, the a priori formal system given in Book I, and its application given in Book II.
This approach is in full agreement with *Tractatus* 6.35:

> Although the spots in our picture are geometrical figures, nevertheless geometry can obviously say nothing about their actual form and position. The network [mesh] is purely geometrical; all its properties can be given *a priori*. Laws like the principle of sufficient reason, etc. are about the net and not about what the net describes.

Hertz's prefatory note to Book I reads:

> The subject-matter of the first book is completely independent of experience. All the assertions are *a priori* judgements in Kant's sense. They are based upon the laws of the internal intuition of, and upon the logical forms followed by the person who makes the assertions; with his external experience they have no other connection than these intuitions and forms have.\(^7\)

In order to show the distinctive contribution of formal structures to our scientific representations of nature, he needs to avoid the mixing of form and content which had plagued the other two formulations. But remember, it is not Hertz's purpose to call into question the suitability of either of the other two pictures for their accustomed purposes. Indeed his aim is precisely to allow us to choose to use them for those purposes with a clear conscience, not distracted by unanswerable foundational questions.

### 7. LINKS BETWEEN HERTZ AND WITTGENSTEIN

It is true that both Hertz and Wittgenstein have "picture theories" in the trivial sense that they use the word *Bild* metaphorically to describe a relation between linguistic and non-linguistic entities. However such a remark is misleading if it is meant to convey any real information about similarities in their thought. For one thing the picture metaphor seems to have been common currency at the time having been used, at least, by Maxwell, Mach, Bolzano and Boltzmann\(^3\). Furthermore the two uses of the metaphor by Hertz and by Wittgenstein are strikingly different in detail. In the *Tractatus* a proposition is a picture of a fact. Hertz, for his part, writes about *two* kinds of pictures, conceptions of things (*Vorstellungen*) and complex scientific representations (*Darstellungen*). Wittgenstein studiously avoids using the word *Vorstellung*, and he explicitly says that scientific theories are *forms* of picturing, that they are *not* themselves pictures and in fact *cannot* be pictures. On the other hand, explaining the nature of these pictorial forms and how they make it possible for us to make pictures of facts *is* central to his project in the *Tractatus*. He explains pictorial form in terms of logical space—a notion which, as we will presently see, has a great deal in common with Hertz's configuration space. Accordingly, the place to look for Hertz's influence is not, as
has usually been done, in the use of the word "pictures," but rather in the twin ideas of pictorial form and logical space.

8. FROM CONFIGURATION SPACE TO LOGICAL SPACE

As mentioned above, Hertz's configuration space is an abstract multi-dimensional space (having arbitrarily many dimensions, as required for individual cases) in which physical processes can be represented. "In this space one point is sufficient to represent the mechanical system and hence we can carry over the mechanics of a free particle to any mechanical system if we place that particle in a space of the proper number of dimensions and proper geometry."\(^{39}\)

In the general case, what is required for this kind of representation is \(a\) a formal structure, which gives the space and its geometry, and \(b\) a method of projection by which the coordinates in the abstract space are mapped onto objects of experience.  The \(a\ priori\) structure of the theory gives the first (as in \textit{Principles}, Book I) and the interpretation and application of the theory to experience gives the second (Book II).  Hertz's formulation of mechanics has the merit of showing that both the formal structure and the method of projection are underdetermined by experience and are subject to free choices that are influenced by specific representational purposes.  In addition, he makes it easier to distinguish 1) the aspects of a particular representational practice which are attributable to the intrinsic formal properties of the space, 2) those attributable to the contingent features of the experiences being represented, and 3) those attributable to the pragmatic features of the mode of projection by which the first makes possible the representation of the second.

Now consider the following remarks from the \textit{Tractatus}:

3.21 The configuration of objects in a situation corresponds to the configuration of simple signs in the propositional sign.

3.4 A proposition determines a place in logical space.

3.411 The propositional sign with logical coordinates—\textit{that} is the logical place.

1.13 The facts in logical space are the world.

2.0131 A spatial object must be situated in infinite space.  (A spatial point is an argument-place).

A speck in the visual field, though it need not be red, must have some color: it is, so to speak, surrounded by colour-space.  [Musical] notes must have \textit{some} pitch, objects of the sense of touch \textit{some} degree of hardness, and so on.
2.0251 Space, time and colour [Farbigkeit] are forms of objects.

2.0272 The configuration of objects produces states of affairs.

2.18 What any picture . . . must have in common with reality in order to depict it . . . in any way at all, is logical form.

5.475 All that is required is that we should construct a system of signs with a particular number of dimensions—with a particular mathematical multiplicity [Mannigfaltigkeit].

These remarks display a development of configuration space into the more general notion of logical space. One of the crucial formal properties of Hertzian configuration space which determined its ability to represent a given physical fact is its particular dimensionality, what he referred to as its multiplicity (Mannigfaltigkeit). In 5.475 the reference is explicit. What are we to make of the idea of the multi-dimensionality of logical space? At least we can say that it is intimately connected with the representational relation between language, thought and the world. Thus we read at 4.04:

In a proposition there must be exactly as many distinguishable parts as in the situation that it represents.

The two must possess the same logical (mathematical) multiplicity [Mannigfaltigkeit].

(Compare Hertz's Mechanics on dynamical models.)

What Hertz says about dynamical models and the structure of thought, in the passage Wittgenstein is referring to here, is fascinating. After explaining the need for dimensional isomorphism between two systems which are models of each other Hertz says:

The relation of a dynamical model to the system of which it is regarded as the model, is precisely the same as the relation of the pictures which our mind forms of things to the things themselves. For if we regard the condition of the model as the representation of the condition of the system, then the consequents of this representation, which, according to the laws of this representation, must appear, are also the representation of the consequents which must proceed from the original object according to the laws of this original object. The agreement between the mind and nature may therefore be likened to the agreement between two systems which are models of one another, and we can even account for this agreement by assuming that the mind is capable of making actual dynamic models of things, and of working with them.40

Hertz leaves this as only a suggestion, an intriguing possibility. But Wittgenstein seems to have taken up the suggestion and developed its implications. This is especially clear in the Tractarian example of the way that a musical thought can be expressed (modeled) in musical notation.41 What dimensionality must the notation have in order to do its job? It must be, we are told, the same as the dimensionality of a performance of the piece as we conceive of it. But what is that? We think of a piece of music as a temporal sequence of sounds (notes) sometimes overlapping and sometimes separated by silences. Each note can be characterized
by such features as pitch, duration, and volume relative to the notes that precede and follow. As a whole, the piece may be characterized by the set of tones that it contains, i.e. its key, and by the tempo at which it is played. In our musical notation, each of these characteristics can be expressed by an independent feature of the notation. The temporal sequence by a spatial sequence, the relative pitch of the notes by their spatial arrangement on the staff, the duration by the shape of the notes, etc.. Most of the dimensions of musical notation are articulated into discrete increments (duration in quarter notes and half notes, tones of the scale as whole steps and half steps). However, some dimensions, for example volume, are continuously variable, as in crescendo and diminuendo. And these features are reflected in the properties of the notation. Taken together they constitute a formal structure with a determinate number of dimensions. Each note can be thought of as occupying a determinate position in that structure, the "musical space." But what allows the structure to represent is not anything inherently musical about the notation; the same notation could be used to represent a train schedule or a menu for a dinner party. That it is musical notation is determined purely by the method used to project its symbols into experiences— the practical ways in which it is learned and used.

Viewed purely as a formal structure, our musical notation merely describes a logical space. Notice that there are potentially expressive properties of the notation—for example the color of the notes and the spacing of the lines of the staff—that are not used in representing features of the musical thought (although we can imagine that they could be). These are the "empty relation" among elements of the representation which, as Hertz noted, cannot be totally avoided. There are also features of the musical performance—such as the size, make and relative location of the instruments—which are not represented in the notation (though, again, they could be). It is fair to say that what counts for us as a musical thought is, in important ways, determined by our accustomed means of representing those thoughts.

It remains to be seen whether this conception of the correspondence between symbols, thoughts and the world can be generalized from the musical example into a satisfying account of all (or even a large portion of) language. That Wittgenstein thought it could is indicated by the fact that he mentions other examples of dimensions of logical space: color-space (as illustrated by the 3-dimensional "color solid") and the hardness scale (like the one rock-hounds use to classify rocks). The suggestion seems to be that all the properties that we attribute to objects in factual statements can be thought of as "living" in some dimension or other of such an abstract space. This is clearly analogous to Hertz's accomplishment of
representing every relevant mechanical feature of a physical system as a dimension in configuration space.

9. PICTURES

We are now in a position to appreciation both the similarities and differences between the two "picture theories." For Hertz, as we have said, "picture" is used to refer either to individual "conceptions" of things or else to whole theories. Theories consist of a priori formal structures together with a mode of interpretation or projection by which they are applied to objects of experience. Within his fully articulated theory, physical facts are represented by points in configuration space. Hertz never makes explicit what justifies the picture metaphor, but his articulated theory, by carefully separating the formal structure from the empirical content makes it possible to see what the essential features of the representational relation are. He provides, in other words, the ingredients for someone else to give a general account of the formal requirements of representation. This, I believe, is the task Wittgenstein set for himself.

Thus in the Tractatus Wittgenstein not only adopts the term "picture", but more importantly, adopts Hertz's attitude toward the structure of representation. Remember that Tractarian pictures are neither conceptions nor formal structures, they are collections of signs (coordinates) which pick out a place in a presupposed formal structure. They depend for their pictorial nature on the formal properties of the structure and on the human purpose, the use, for which that structure is employed. These purposes are displayed in the interpretations or methods of projection which are part of the theory.\textsuperscript{42} Theories are not pictures but the forms of pictures, and they only produce pictures relative to someone's representing intention. In the net analogy we saw Wittgenstein's suggestion that human purposes determine the choice of which form to use and thus the level of detail of the picture. In the musical example we saw how the structure of the situation being depicted put certain constraints on the structure of the notation, again relative to a particular purpose.

This then is another generalization on Hertz's point, that when we select a theory or other representational practice, we do so because it is suitable to our purposes (zweckmäßig), because it meets our requirements (Forderungen). If one of those requirements is of a logical nature, we can expect that there will be corresponding logical constraints put on our choice of representational forms.
Let us then recall what Hertz had called the "fundamental requirement" on all our representations of nature. It is an inferential requirement: that "the consequents of our pictures be pictures of the consequents". This has the ring of a slogan, and a very odd one at that. If we are to take it seriously, our first question ought to be: how can pictures have consequents? What, that is to say, is the connection between logic, the formal analysis of patterns of inference, and the distinctively human practice of making representations? It is my principal thesis that Wittgenstein, in trying to understand Hertz, took this slogan and the question it raises very seriously. In the *Tractatus*, he gives the following answer to the question "when can pictures have consequents?" His answer is: when those pictures are propositions which contain or share certain formal properties which allow us to draw inferences from them. This, then is the justification for calling abstract representational spaces "logical spaces." Their structure is what allows us to draw inferences from facts expressed using them.

Wittgenstein had inherited, from Frege and Russell, powerful tools with which to analyze what this Hertzian inferential requirement amounts to. To begin with, since inferential relations are relations among the truth values of sentences, truth values must be assignable to points in logical space. The link between a proposition and its truth value is its sense. Within a system of propositions if we require inferential relations we *ipso facto* require that individual propositions have determinate truth values, and this, in turn requires that they have determinate senses. If a proposition has a determinate sense, it must be because the method of projection by which we map its representational space onto experience allows us to determine its truth value. This is the reason that Tractarian propositions *must include* their projective relation to the world (3.12). If there is no interpretive practice which dependably assigns truth-values to points in logical space, there are no propositions. If there *is* such a practice, there is automatically a whole system of propositions corresponding to those points. In the application of our method of projection we hold the system of propositions up against reality like a ruler" and determine which of its members is true.

Wittgenstein then draws further consequences from the fundamental requirement. At 3.23 we read "The requirement (*Forderung*) that simple signs be possible is the requirement that senses be determinate." What is the significance of this remark? It is one of a number of initially puzzling statements of the form "A is B" which occur at key points in the *Tractatus*. In this case we are given an equivalence between two requirements.
Understanding what is meant by one side of the equivalence should help us to understand what is meant by the other. Let us begin with the second half. For a proposition to have a determinate sense means for it to unambiguously determine a point in logical space. In order for it to do this, the space must have a definite geometry and the propositional sign must contain the coordinates proper to that space. The proposition must also include a projective relation which is capable of assigning a definite truth value to that point. Within that space and relative to that method of projection, those coordinates which function in the determination of that point and its truth value have a special status. Wittgenstein identifies them as the "simple signs" which occur in propositions that are fully analyzed; he says they stand for simple objects in the states of affairs which those propositions picture. So what 3.23 is telling us is that when we require that propositions have determinate senses—that it be possible to determine their truth values so that we can use them to draw inferences—the nature of representation dictates that we are simultaneously requiring that there be a level of analysis at which the "distinguishable parts" of our symbolism match up one to one with the dimensions of the logical space being used. Reflecting on the net analogy and the musical notation example, along with the "methods of projection" implicit in them, should make this clear. Here we have a direct connection between the simplicity of objects and "the requirement that senses be determinate" and thence to the general requirement that we be able to make inferences from some factual statements to others.

Is the requirement that some objects be simple a kind of ontological dogmatism, or Tractarian mysticism? At this point we would do well to recall the suggestion of ontological relativism which we noted in Hertz's discussion of choices among alternative theories and which Wittgenstein echoes in the net analogy when he talks about using various nets to describe the same state of affairs.

11. OBJECTS, FACTS AND PICTURES

We noted in connection with the net analogy that factual discourse requires selection of some representational form or other, but does not prescribe any particular form. Specifically the net analogy suggests two kinds of choices, the shape of the net and the fineness of the mesh. Both of these choices have consequences for the ontology of the description. The shape of the grid determines what kinds of objects the description will be made in terms of (c.f. Hertzian vs. Newtonian descriptions of planetary motions). The fineness of the mesh determines a depth of analysis (in Hertz's mechanics, the choice of which connections to treat
as rigid). The grid need not have any particular shape or size, but it must have some definite size, some definite shape, or other. This is the choice that determines what the simple objects are. These objects are simple relative to the choices we make about the pictorial forms we use, and these choices, in turn are relative to our purposes. In making the choices which determine the simple objects, we are also making a choice which determines what the most basic combinations of those objects will be, thus what the facts will be which we will take into account when assigning truth-values to points in logical space.

The way Wittgenstein thinks this works is clear from the net analogy. If your job is to give a description of the shape of an inkblot on a surface, you can do it by dividing the surface into parts in a formal way and indicating which parts are covered by the blot. Your choice of the system of division will depend on how much detail you need to show. No matter how fine-grained your division, there will always be some distortion along the boundary of the blot. But an infinitely fine division is not an option. Such a requirement would defeat the whole project. But given any reasonable reproductive purpose, you can find a level of division that will get you as close to the boundary as you want to be (beliebig nahe). In choosing a specific level of detail, one chooses the lowest level of information available for representation by the system, the simplest fact that the system can recognize and express, and thus the simple signs and the simple objects relative to that system.

12. RELATIVE SIMPLICITY OF OBJECTS

The plausibility of this notion of relative simplicity of objects depends upon the distinction between the form and the content of a system of representation. We have seen the importance of this distinction for Hertz, and Wittgenstein’s remarks in the Tractatus about formal concepts are based on that distinction. In the net analogy this distinction is apparent in the fact that a net of a given shape and fineness can be used to describe an inkblot of any shape, but it cannot be used to describe itself (although it could obviously be described by another, finer net). The shape and size of the inkblot is the content of the representation. The shape and size of the individual unit of the net is its form and can never become part of the content. What the analogy shows is that the concept of an object is something that is presupposed by every representational practice in the same way that the concept of force is presupposed by Newtonian mechanics. In neither case can these concepts, which are essentially bound up with the form of representation, become part of the content represented. In the Newtonian system of representation we use the concept of force to answer questions
like "What is the orbit of a planet?" but we cannot use the same system to answer the question "What is force?" A parallel violation of the form/content distinction makes the question "What is an object?" an illegitimate question in the same sense as "What is force?"

Losing sight of the form/content distinction has been a source of confusion among critics of both Hertz and Wittgenstein. Peter Barker, for example, has suggested a link between Hertz's concept of material points and the simple objects of the *Tractatus*. Material points are introduced by Hertz as part of his *a priori* definition of mass in Book I. He stipulates that they be conceived as made up of an infinite number of infinitesimal material particles, and that the mass of a material point is a numerical quantity which represents the ratio between the number of material particles in two material points, one of which is chosen as a standard. This allows mass to take any real value, rational or irrational. It also makes the question of the mass of a material particle illegitimate ("metaphysical", says Barker). These definitions are part of the formal structure of the theory and are designed to fulfill specific purposes. One purpose is that the theory provide definite numerical values to attach to its basic concepts without prejudging the continuity or discontinuity of the subject matter to be represented. Barker calls this a "curious metaphysical dodge" and complains that it avoids embarrassing questions about the nature of material particles. But the illegitimacy of such questions is just the point of introducing the definition in the first book, they are part of the form of the representation and cannot become its subject matter.

Wittgenstein is then accused by Barker of following Hertz maneuver in introducing simple objects in the *Tractatus*. Wittgenstein claims that in order for propositions to have determinate truth values, there must be a lowest level of logical analysis, there must be elementary propositions, and their components must correspond to simple objects. But the nature of these objects is beyond the expressive power of language. Barker suggests that the unrepresentability of objects repeats Hertz trick for avoiding giving a straight answer about his basic concepts. There is certainly a parallel but not the one Barker sees. Like "mass" In Hertz's theory, "object" in the *Tractatus* is a formal concept, it is part of the *a priori* structure of a system of representation. To say that there are simple objects or that mass can take any positive real value is not to make any factual claim about the physical indivisibility of any physical object (whether quarks, iron bars, planets or tables); nor is Hertz claiming that science will never find a smallest unit of matter. In both cases the statements are not dogmatic, agnostic or metaphysical, they are *formal*. They are meaningful relative to a signifying structure of which they are a part, and they have practical significance only relative to some representational purpose for which that signifying structure might be adopted.
Tractarian objects are simple relative to a particular application of the descriptive structure of language, namely in making factual statements which can enter into inferential relations with one another.

Besides Hertz's 'material points' here are other examples of the idea of relative simplicity with which Wittgenstein was familiar. For example Frege, in *Foundations of Arithmetic*, §54, notes that finite numbers require for their application definite units, concepts under which objects either fall or do not fall without vagueness or ambiguity: "Only a concept which isolates what falls under it in a definite manner, and which does not permit any arbitrary division into parts, can be a unit relative to a finite number." It should be noted that "indivisibility" here has a special meaning. Numbers can only be sensibly assigned to concepts which divide the world in definite ways, so he suggests thinking of units as indivisible (simple) relative to those concepts. The word "house" has the number one sensibly applied to it under the unit "syllables in the word x" and has the number 5 applied to it under the unit "letters in the word x". Both units are definite and indivisible, but the indivisibility of the word relative to one unit does not imply indivisibility relative to others.46

13. HERTZ'S INSIGHT, GENERALIZED

In doing physics the scientist constructs systems of representation within which facts can be described. He operates within both logical and practical constraints. The system of representation can be constructed *a priori*; some might wish to say that an infinite number of such systems have always existed, just waiting to be discovered. But since any such system is a form within which a given set of facts may or may not fit conveniently, it is the scientist's job to develop the projective relationship between such a form and the reality it is used to represent. This projective relationship will always depend on some kind of institutionalized human activity (e.g., reading clocks, scales and rulers) to bridge the gap between the form and the content. Thus the pure form is interpreted by showing how it connects with experience. But the choice of when and how to use a theory is always relative to a set of human purpose which are not, themselves subject to theoretical expression within that theory. In giving us this description of scientific practice Hertz "draws the limits of theoretical representation from the inside." I believe that one of Wittgenstein's principal innovations in the *Tractatus* was to extend this insight to cover factual uses of language in general. In recognizing this we recognize the nature, and the extent, of Hertz's influence on that work.

Joseph Tougas
University of California, Irvine
[jtougas@ea.oac.uci.edu]

* A shortened version of this paper was presented at the Pacific Division meeting of the APA, March, 1998. I thank C. G. Luckhardt for his comments at that session. For comments on earlier versions I thank participants in the Philosophische Gesellschaft in Salzburg, Austria, especially Edgar Morscher, and Otto Neumaier and Johannes Brandl, and also, at Irvine, David W. Smith, Penelope Maddy, Alan Nelson and Jeff Barrett. A German version of some of this research has been published, in connection with the Philosophische Gesellschaft presentation, in Conceptus, XXIX, 1996, pp. 205-227.

REFERENCES

(The "Einleitung" to Prinzipien has been reprinted as vol. 263 of the series Ostwalds Klassiker der exakten Wissenschaften., akademische Verlagsgesellschaft, Geest & Portig, Leipzig, 1984.)
(D. F. Pears and B. F. McGuinness translation, 1961, same publisher)

NOTES

1 This passage originates in the Wittgenstein Nachlass, manuscript 154, p. 43.
3 Wittgenstein, A Life. p. 39. The quotation is from Philosophical Grammar, §89.
4 For a detailed exposition of the philosophically relevant issues in theoretical physics during the last decades of the 19th C. see Ernst Cassirer, (1950, Ch. V). See especially the description of Hertz crucial contributions, pp. 103-109. This is the clearest exposition I have found of Hertz's philosophical views and their influence, though Cassirer does not mention the Tractatus. A brief historical sketch given by Peter Barker in "Hertz and Wittgenstein," (1980, pp. 244-247) is more narrowly targeted at the aspects of Hertz's work most relevant to our concerns here.
5 Cassirer (1950, p.87).
7 Cassirer (1950 p. 106).
9 The Variational Principles of Mechanics (1966, Ch X). Lanczos gives a detailed picture of the turmoil in the foundations of physical theory, and of mechanics in particular, to which Hertz's Principles was a response. He also explains the intricacies of the rival formulations which Hertz considers there. Such information is indispensable, not only for understanding what Hertz is talking about, but also for understanding the enthusiasm and admiration that his work seems to have evoked in many of his readers, including the young Wittgenstein.
10 "Wir machen uns innere Scheinbilder oder Symbole der äusseren Gegenstände." c.f. Tractatus, 2.1: "Wir machen uns Bilder der Tatsachen." The contrast between the two formulations is highly significant, and making this contrast understandable will be a central goal of the present essay. At this point it will suffice to point out three salient differences: a) the lack of contrast in the Tractatus formulation between the inner nature of the picture and the outer nature of what is pictured, b) Wittgenstein's replacement of Hertz's vague and tentative "Scheinbilder oder Symbole" with the straightforward "Bild" and, c) the change in what is being pictured, from Hertz's "Gegenstände" to Wittgenstein's "Tatsachen."
11 "Die Bilder ... sind unsere Vorstellungen von den Dingen."
12 Generally translated "agreement," this is the word that Wittgenstein uses at 2.21 to explain the truth of pictures.
13 "Die denknotwendigen folgender Bilder stets wieder die Bilder seien von den naturenotwendigen Folgen der abgebildeten Gegenstände." Note the parallel and contrast between cognitively necessary inferences (which Hertz later links with logical necessity) and the physical necessity which he supposes to be operating in nature. Wittgenstein's reaction to this is visible at 6.375, where he says "all necessity is logical necessity." Note also Hertz use of "abbilden" to characterize the relation between ideas and things.
14 Usually translated "image" by Jones and Walley in the English version.
15 Janik and Toulmin choose to translate it as "model" in all quotations from Hertz and Wittgenstein. This is not only indefensible, since both authors use the German cognate Modell in other places to talk about just the kinds of things we normally call models, but it conceals just the metaphoric suggestiveness that is crucial to the exploratory thinking that both authors are engaged in.
16 How, it has been asked, can this be a meaningful requirement for a scientific theory? Since it is supposed to rest on "the laws of thought," a logically impermissible theory would not even be thinkable (cf.
Wittgenstein's remark about the impossibility of thinking the illogical). But this is just a mistake about Hertz's meaning, perhaps provoked by the tendency of contemporary philosophers to equate logical permissibility with consistency. Peter Barker, e.g., does this (1980, p. 245). In the Tractatus (5.451, f.) Wittgenstein worries about the same problem of equivocation arising in relation to the introduction of primitive symbols in logical systems. The clarity of the theory is compromised by failure to keep the articulation of the logical structure separate from its interpretation and application.

17 Peter Barker (1980) translates it "perspicuity." The inadequacy of this becomes obvious on p. 250, where it results in the following: "The perspicuity of which we have spoken has no reference to the practical applications or the needs of mankind. In respect of these latter it is scarcely possible that the usual representation of mechanics, which has been devised expressly for them, can ever be replaced by a more perspicuous system."

18 P. 40. On the appropriateness of Newtonian mechanics to its original set of problems, see p. 10. The way Hertz organized the "Introduction" has led some commentators to conclude that the description of Zweckmäßigkeit, presented there is intended as a standard for all scientific theories. But other statements elsewhere in the Principles make it clear that for some purposes lack of clarity and simplicity ought to be tolerated in return for other gains, pragmatic, pedagogical, etc..

19 Hertz sets for himself the further requirement that the exposition of his theory be arranged in such a way that it makes apparent which features of the representation are determined by the need for logical permissibility, which for the sake of correctness and which for the sake of appropriateness. It is the latter which are unconstrained either by "the nature of our minds" or by the "facts of experience (Erfahrungstatsachen) which have served in the construction of the picture," and which are therefore subject to our free choice. These arbitrary elements, the symbolism, definitions, abbreviation, etc., are the ones that we can manipulate in order to achieve new and different pictures which may prove better suited to various purposes, both new and old.

20 Jeff Barrett has helped me clarify the formulation of the next two sections. My aim here is to portray as accurately as possible Hertz's perspective and concerns while recognizing subsequent refinements and interpretations of the theories he discusses.

21 Later developments in physics have allowed the unification of forces. This, however, only confirms Hertz's point that what is now meant by "force" is not what was meant previously.

22 P. 11-12 in the Dover edition (Because of the translational difficulties noted above, I have provided my own translations of all passages, though I follow Jones and Walley when possible). It is in this connection that Hertz introduces the metaphor of the "idle wheels" (leergehende Nebenräder, which Jones and Walley translate, unhelpfully, as "sleeping partners.") On the connection of this metaphor with similar ones in Wittgenstein see Barker (1980, 243-256).

23 Principles, p. 13. We are not, it must be said, the unprejudiced persons Hertz asks for. We are so accustomed to switching back and forth between what Wilfrid Sellars has called the scientific image and the manifest image of the world that we can, perhaps, only recover the phantasmatic flavor of theoretical world pictures by attending carefully to what it feels like to move between classical, relativistic representations of familiar phenomena such as simultaneous events and motion in a gravitational field.

24 Lanczos (1966, p. 27) describes this shift in primitive conceptions from force to work as follows: "Although we are inclined to believe that force is something primitive and irreducible, the analytical treatment of mechanics shows that it is not force but the work done by the force which is of primary importance, while the force is a secondary quantity derived from the work."

25 Helmholtz, for instance, had shown how this approach could account directly for some phenomena for which classical physics had needed to posit separate forces.

26 Hertz has in mind the Kantian notion of substance—that which remains (is conserved) through change. Thus the law of conservation of energy makes a commitment to the substantiality of energy. This is also the notion of substance operative in the Tractarian doctrine that objects are the substance of the world. Note also Tractatus 6.3ff. esp. 6.33, 6.34, 6.35 and 6.36. As we shall see, both for Hertz and for Wittgenstein, substance is a formal concept, and thus a link between forms of representation and ontological commitments.

27 "Every system of natural bodies moves just as if it were assigned the problem of attaining given positions in given times, and in such a manner that the average over the whole time of the difference between the kinetic and potential energy shall be as small as possible." Principles, p. 16.

28 Ibid., p. 23.
Here is a first hint of the influence Hertz seems to have had on Wittgenstein's thinking about issues of simplicity and complexity in our representations of reality. Cf., *Tractatus*, 4.2211: "Even if the world is infinitely complex . . . there must still be objects and states of affairs." On the Tractarian view of the structural parallel between statements and states of affairs, some of these objects must be "simple." The reason for this puzzling claim, we are told, is that, in requiring statements about the world to have determinate senses and truth values we require that some objects be simple. The relation between our requiring simplicity in our representations and the resulting "relative simplicity" of Tractarian objects will be discussed in detail below.

Cf. *Tractatus* 6.3611, which continues the thought of 6.361 by showing how we use a formal structure such as a system of temporal designations to make our world *gesetzmäßig*: "We cannot compare a process with 'the passage of time'—there is no such thing—but only with another process (such as the working of a chronometer)."

The following technical details will be helpful in understanding the relationship, described below, between Hertz's configuration space and Wittgenstein's notion of 'logical space.' Multi-dimensional representation systems like Hertz's allow the compact expression of great amounts of information. The position of a single particle can be represented by three coordinates in a three dimensional rectangular coordinate space. The positions of two particles can be represented either by two triples of coordinates which pick out two distinct points in three dimensional space or by a sextuple which picks out a single point in six-dimensional space. Generally, any system of N particles can be represented as a single point in 3N dimensional space. Hertz also recognized that by using a slightly different geometrical technique called *general coordinates* the same could be done for the positions of rigidly connected bodies in spaces of less than 3N dimensions.

But the geometry of spaces for systems generated using general coordinates are not, in general, Euclidian—they take on a 'curvature.' To deal with this, Hertz supplements this representational technique with an ingenious application of Riemannian geometry. Riemann had shown that certain topological features of space were preserved when coordinate representations were mapped from one kind of coordinate space onto another, for example, from a flat surface onto a spherical or other curved surface. A central concept in Riemannian geometry is the "line element." By identifying segments of the paths of moving points ("path elements") with Riemannian line elements, Hertz was able to map the motion of mechanical systems into higher dimensional coordinate spaces in which certain formal relations were preserved. (The relative positions of the represented bodies he called "configurations," and so the high dimensional spaces in which they are represented are customarily referred to as "configuration spaces.") Given the correct "mapping" or "projection" of the motion of bodies into configuration space, Gauss's principle of least constraint becomes equivalent to the geometrical requirement that the point which represents the configuration of a mechanical system moves along a path which is as straight as possible, given the geometry of that space. Hertz then refined and developed this technique into a complete theory of mechanics.

"This mathematical mode of expression enables us to state the fundamental law. Without this we should have to split it up into Newton's first law and Gauss's principle of least constraint. Both of these taken together would represent the same fact, but in addition to these facts, they would by implication contain something more, and this something more would be too much." *Principles*, p. 31.

In this terminology I follow Ulrich Majer, who gives an insightful reading of this passage, and its relation to Hertz's philosophy of science (see Majer, 1985, pp. 59-60). Majer also argues (in 1983 and 1985) that Hertz and Wittgenstein share a kind of "conventionalism" in their philosophies of science—a position closely related to the "ontological relativity" which I discuss below.

All quotations from the *Tractatus* are based on Pears and McGuinness. Alterations in the translation are noted and explained. Here they translate *eine Form* as "one form" but the implication of a single acceptable form seems unwarranted by the context. Thus, I follow Ogden.

My use of the term 'ontological relativity' is obviously anachronistic, a perhaps overly provocative; the point here is not to suggest an anticipation of Quine, but rather to highlight a striking historical example of a practicing scientist reflecting on the possibility of what would later be called alternative "conceptual schemes" or alternative "scientific paradigms." What is particularly relevant here, and emerges even more clearly in Wittgenstein, is the connection between the *ontological commitments* of alternative theoretical descriptions and the *representational forms* by means of which those descriptions are given. These
intimations of ontological relativity and the distinction between properties of the net and properties of that which the net describes will come into sharp focus when we look at the Tractarian doctrine of simple objects.

37 P. 45.

38 See Andrew Wilson (1989, pp 250, ff.).

39 Principles, p. 22. For the generalization from 3N-dimensional rectilinear space to n-dimensional Riemannian space by the use of general co-ordinates see p. 24. Hertz introduces both co-ordinate systems as permissible representations in his system.

40 Principles, p. 177.

41 4.01-4.0141. Wittgenstein returned throughout his career to the example of musical notion as an intuitive illustration of the detailed structure of the relation between language and non-linguistic reality; the use of, and learning of musical notation figure prominently in Wittgenstein's Cambridge lectures, and constitute a clear forerunner of "language-games."

42 Cf. 3.12: "A proposition is a propositional sign in its projective relation to the world." and 3.11: "The method of projection is to think of the sense of the proposition." See, also 4.0141 re. "the law of projection of musical thoughts into notation and recordings." Merrill and Jaakko Hintikka have provided important insights into an important link between the "picture theory" of the Tractatus and the later notion of "language games" by tracing Wittgenstein's recurrent and interrelated remarks about the application (Anwendung) of language in our lives, the projection of language on the world, and the use, or uses of language. See Hintikka and Hintikka, Chapters 8 and 9.

43 This is the formulation of 2.1512 ([A picture] is laid against reality like a ruler) which Wittgenstein later said he wished he had used. Cf., e.g., the conversation with Waismann, Dec. 25, 1929, published in Philosophical Remarks, Appendix B, p.317.

44 Principles, p. 46.

45 This is how we should understand Wittgenstein's remark, reported by Norman Malcolm (1958), that at the time he wrote the Tractatus he felt no need to give examples of simple objects because "he was a logician; and that it was not his business, as a logician, to try to decide whether this thing or that was a simple thing or a complex thing, that being a purely empirical matter!"

46 Cf. also Wittgenstein's struggle with the notions of simple signs and simple objects in the Notebooks, 1914-1916, beginning on June 14, 1915. Of special interest in understanding the net analogy is the remark on, June 16, that "a proposition may indeed be an incomplete picture of a certain fact but it is always a complete picture. . . . Let us assume that every spatial object consists of infinitely many points, then it is clear that I cannot mention all these by name when I speak of that object. Here then would be a case in which I cannot arrive at the complete analysis in the old sense at all; and perhaps just this is the usual case. But this is surely clear: the propositions which are the only ones that humanity uses will have a sense just as they are and do not wait upon a future analysis in order to acquire a sense." Tractatus 3.23 occurs in the entry for June 18. Also in that entry: "If I say, e.g., that this watch is not in the drawer, there is absolutely no need for it to FOLLOW LOGICALLY that a wheel which is in the watch is not in the drawer, for perhaps I had not the least knowledge that the wheel was in the watch, and hence could not have meant by 'this watch' the complex in which the wheel occurs." This train of thought culminates in the entries for June 21st and 22nd, 1915: " . . . The simple sign is essentially simple. It functions as a simple object. . . . Logic as it stands, e.g., in Principia Mathematica, can quite well be applied to our ordinary propositions, e.g., from 'All men are mortal' and 'Socrates is a man' there follows according to this logic 'Socrates is mortal' which is obviously correct although I equally obviously do not know what structure is possessed by the thing Socrates or the property of mortality. Here they just function as simple objects. . . . So we see that this simplicity is only constructed. . . . Now when I do this and designate the objects by means of names, does that make them simple? But plainly (aber doch) this proposition is a picture of that complex. This object is simple for me. . . . Only the complex part of the proposition can be true or false. The name compresses its whole complex reference into one."

29