Moran, Chapter 2 – solutions

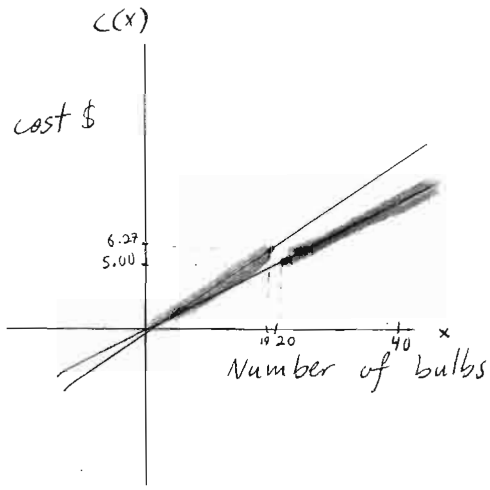
(2) Given:

$.99/3 bulbs when you buy less than 20
$1.00/4 bulbs when you buy 20 or more

a) $C(x) = mx + b$. $m = .99/3 = .33$. $b = 0$ because the cost for 0 bulbs is 0. So $C(x) = .33x$

b) $C(x) = mx + b$ $m = 1.00/4 = .25$ $b = 0$, so $C(x) = .25x$

c)



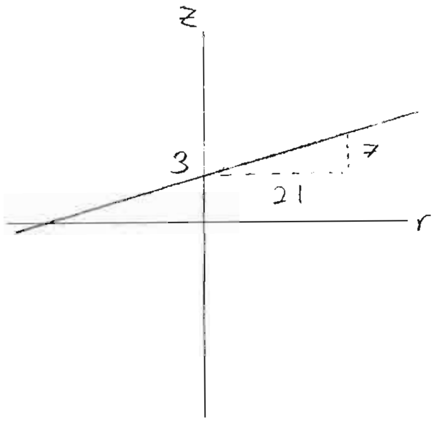
(5) Given:

resting heartrate: 50 bpm
heart rate at 6mph: 125 bpm

a) $y = mx + b$, where x is running speed. When $x = 0$, $R(0) = 50$ (resting heart rate), so $y = mx + 50$. To find m, use $\frac{y_2 - y_1}{x_2 - x_1} = \frac{125 - 50}{6 - 0} \approx 10.83$
So $R(x) = 10.83x + 50$. The first constant, $m =$ 10.83, is increase in heart rate for each mph of running speed, in bpm/mph. The second constant, $b = 50$, is the resting heart rate in bpm.

b $R(9) = 10.83(9) + 50 = 147.5$ bpm

(10)



$\Delta y = 7$ and $\frac{\Delta y}{\Delta x} = m$ (slope)
$\Delta x = 21$
$b = 3$ (y-intercept)

So $z = \frac{7}{21} r + 3 \rightarrow z = \frac{1}{3} r + 3$

(14) Given:
The elevation of Troy rises at the rate of 4.7 ft./century
The ground level of Troy in 1500 BC was approx. 123 ft. above sea level.

a) 3000 BC is 15 centuries before 1500 BC. The ground level in 3000 BC is $123 - 15(4.7) = 52.5$ ft.

Today is approx. 25 centuries after 1500 BC. The ground level today would be $123 + 25(4.7) = 240.5$ ft.

20 centuries ago was approx. year 0 AD. This was 15 centuries after 1500 BC. The ground level then was $123 + 15(4.7) = 193.5$ ft.

b) In our standard equation $y = mx + b$ we know the rate of change, $m = 4.7$ ft./century. We also know that the height at year 0 was 193.5 ft. So our function is

$$H(t) = 4.7t + 193.5$$

where t is time, in centuries, since the year 0 AD.

(17)

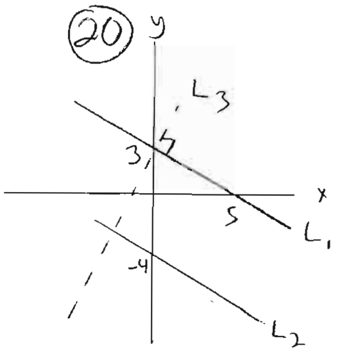
$T(2) = 11 \qquad T(5) = 6 \qquad T(12) = -4$

$$\frac{T(5)-T(2)}{5-2} = \frac{6-11}{5-2} = \frac{-5}{3}$$

$$\frac{T(12)-T(5)}{12-5} = \frac{-4-6}{7} = \frac{-10}{7}$$

between 2 and 5, the rate of change is $\frac{-5}{3}$ and between 5 and 12, the rate of change is $\frac{-10}{7}$. The rate of change is not constant, so this cannot be a linear function.

(20)



a) $y = mx + b \rightarrow b = 3 \rightarrow y = mx + 3 \rightarrow m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-3}{5-0} = \frac{-3}{5}$
So $L_1 = \frac{-3}{5}x + 3$

b) L_2 has the same slope as L_1, and y-intercept -4,
So $L_2 = \frac{-3}{5}x - 4$

c) The slope of L_3 is the negative reciprocal of the slope of L_1 and L_2, and L_3 has y-intercept 3. So $L_3 = -\left(-\frac{5}{3}\right)x + 3 = \frac{5}{3}x + 3$.

(21)

a) In this model, the population doubles every 50 years. This is an exponential growth rate.

b) $P(x) = Ca^x$ $C = 1993$ population $= 5.5$ billion, a = growth factor = "double" $= 2$. x = time, in 50-year periods since 1993. Putting it all together: $P(x) = 5.5\text{billion}\,(2^x)$.

c) The population will quadruple when $2^x = 4$ — when $x = 2$
2 50-year periods = 100 years

The population will be 16 times its 1993 level when $2^x = 16$ — when $x = 4$. 4 50-year periods = 200 years.

d)

We want to find the annual growth factor, a. Notice that

$$5.5 \text{ billion } (2^1) = 5.5 \text{ billion } (a^{50}), \text{ so } 2 = a^{50}, \text{ and therefore,}$$

$$2^{1/50} = (a^{50})^{1/50} \rightarrow 2^{1/50} = a \approx 1.014$$

This growth factor is similar to the one used on p. 66

e) Eventually we will run out of planet

27

Given:
Initial population: 24 deer in 1990
growth rate: 29% /year

a) In 1991 there were 29% more deer:
$24 + 24(.29) \approx 31$ deer
In 1992:
$31 + 31(.29) \approx 40$ deer

b) $D(t) = Ca^t$, where t is years after 1990.

C = Initial population $= 24$
a = growth factor $= 100\% + 29\% = 129\% = 1.29$.

So $D(t) = 24(1.29)^t$

d) Using a graphing calculator: about 8 years.

(30) Given:

2-week periods	Snails
0	8
1	20
2	50

Initial population: 8 snails — $N(0) = 8 = C$
2 weeks later: 20 snails — $N(1) = 20$
4 weeks later: 50 snails — $N(2) = 50$

a) Notice (as above) that t is in 2-week periods.
To find the growth factor, use $a = \frac{y_2}{y_1} = \frac{20}{8} = 2.5$

So $N(t) = Ca^t = 8(2.5)^t$

b) Suppose you use 50 for y_2 and 8 for y_1. Then the growth factor is $\frac{50}{8} = \frac{25}{4} = 6.25 \doteq 2.5^2$.
6.25 is the correct growth factor for a 4-week period, but it is not correct for a 2 week period.
To regain the correct growth factor, notice that $6.25 = 2.5^2$. So in this case, $a = 6.25^{1/2} = 2.5$

c) Notice that the unknown growth factor, a, has the property that $a^{14} = 2.5$, because 14 days = 1 2-week period.
So $a = 2.5^{1/14} \approx 1.067$. The new equation, with t in days, is

$$N(x) = 8(1.067)^x$$

(34)

From the graph we can construct the following table to find the growth factor a:

period x	value y
0	1800
1	?
2	?
3	500

$\frac{500}{1800} = a^3$, since $f(0) = 1800$ and $f(3) = 500$. $f(0)$ and $f(3)$ are not consecutive values.

$$\frac{500}{1800} = a^3 \rightarrow \frac{5}{18} = a^3 \rightarrow \left(\frac{5}{18}\right)^{1/3} = a \rightarrow a \approx .652$$

The graph also tells us the y-intercept, C, of the function: $C = 1800$. So $y = Ca^x = 1800(.652)^x$

38

Given:

x	0	1	3	5
y	10	50	1250	31250

$\frac{50}{10} = 5, \quad \frac{1250}{50} = 25 = 5^2, \quad \frac{31250}{1250} = 25 = 5^2$

Between $y(0)$ and $y(1)$, the constant of proportionality is 5. Between $y(1)$ and $y(3)$, and $y(3)$ and $y(5)$, the constant is $25 = 5^2$, and 1, 3 and 3, 5, are a distance of 2 apart, which is consistent with an exponential model with growth factor $a = 5$.

Since $y(0) = 10$, 10 is the y-intercept/initial amount,

So $y = 10(5)^x$ is a possible formula for this function.