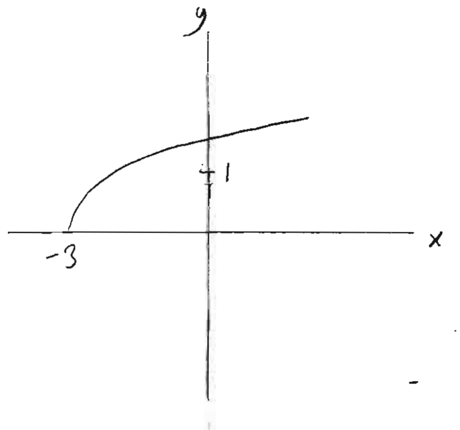
Precalc Solutions, Wk 8

Chapter 3

② Identify the family for each graph and write an algebraic function.

a)

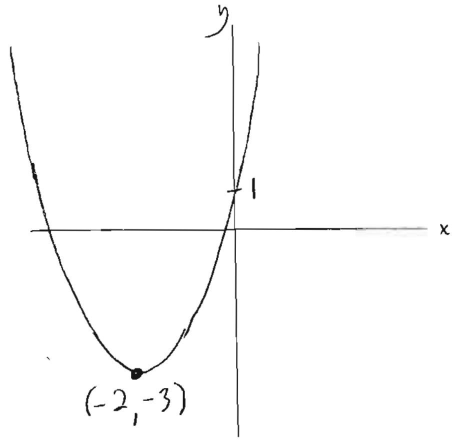


family: Square root: $y=\sqrt{x}$

formula: $y=\sqrt{x+3}$

(Horizontal shift 3 units to the left)

b)

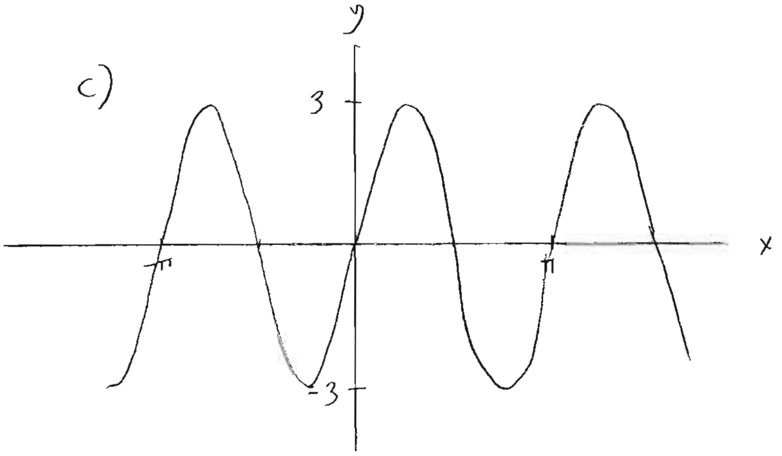


family: square: $y=x^2$

formula: $y=(x+2)^2-3$

(Horizontal shift 2 units left and vertical shift 3 units down)

c)

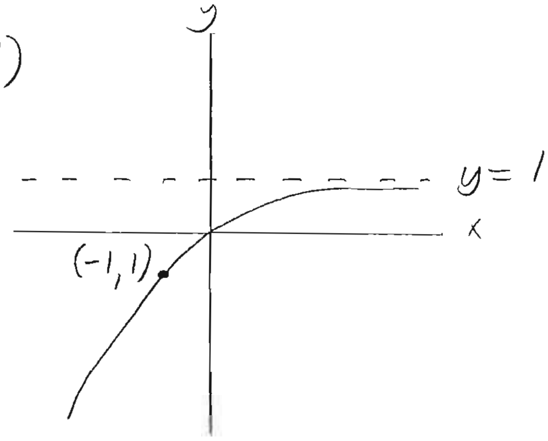


family: sinusoidal: $y=\sin x$

formula: $y=3\sin x$

(Vertical stretch by a factor of 3)

d)

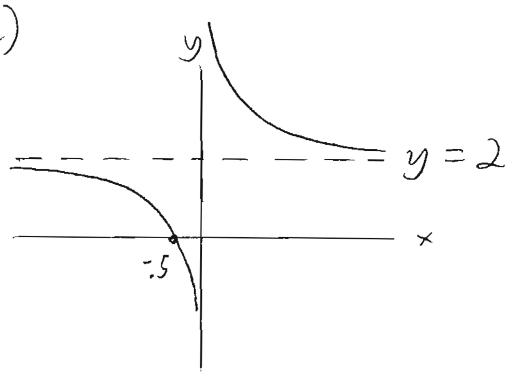


family: exponential: $y = a^x$, $0 < a < 1$

formula: $y = -.5^x + 1$

(Vertical flip over x-axis and vertical shift up by one unit. The growth factor is found by solving for a: $-1 = -a^{(-1)} + 1 \rightarrow a^{-1} = 2 \rightarrow \frac{1}{a} = 2 \rightarrow a = \frac{1}{2}$)

e)

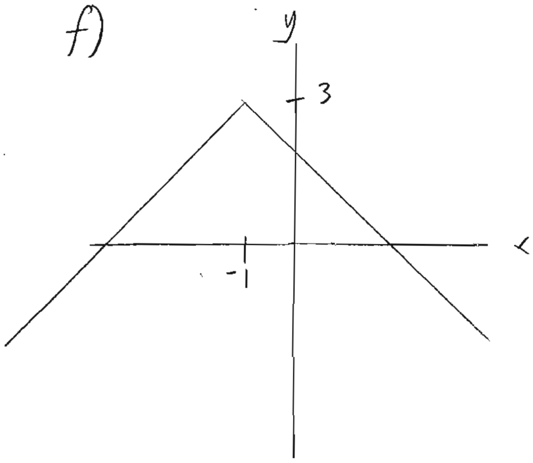


family: reciprocal: $y = 1/x$

formula: $y = \frac{1}{x} + 2$

(Horizontal shift 3 units up)

f)



family: absolute value: $y = |x|$

formula: $y = -|x+1| + 3$

(Vertical flip over x-axis, Horizontal shift 1 unit left, vertical shift 3 units up.)

④ Let $g(x) = 10^x$. Write the formula that results when the graph of g is:

a) Shifted 3 units right:

$$g(x) \to g(x-3) \to 10^{x-3}$$

b) Shifted 3 units up:

$$g(x) \to g(x) + 3 \to 10^x + 3$$

c) Reflected in the x-axis:

$$g(x) \to -g(x) \to -10^x$$

d) Reflected in the y-axis:

$$g(x) \to g(-x) \to 10^{-x}$$

e) Compressed toward the x-axis by a factor of 3:

$$g(x) \to \frac{1}{3} g(x) \to \frac{1}{3}(10^x)$$

f) Compressed toward the y-axis by a factor of 3:

$$g(x) \to g(3x) \to 10^{3x}$$

(12) "The population of Parallel doubles every 30 years. Its current population is 50,000."

a) Write a mathematical model for the population of Parallel as a function of time after now, with time in 30 year periods:

Growth rate: 2, Starting amount: 50,000

$P(t) = 50{,}000\,(2^t)$

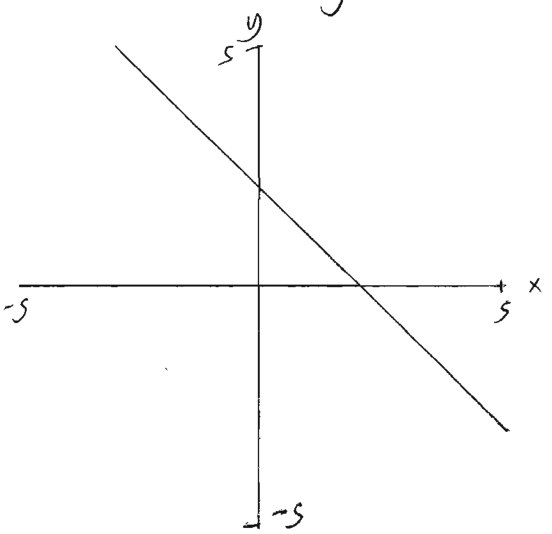
b) Use a stretch to rewrite it in years:

1 30-year period = 30 years, so

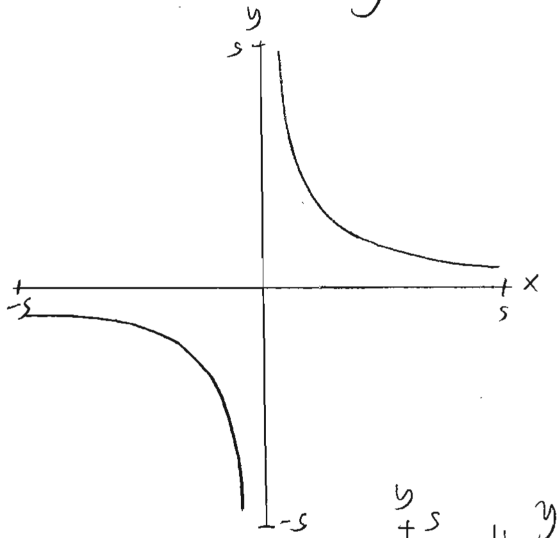
$P(t) = 50{,}000\,(2^{t/30})$.

(16) a) Estimate:

$y = h(x)$

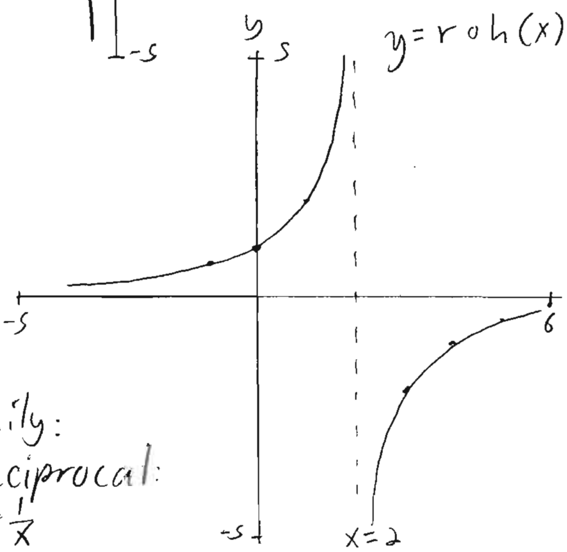


$y = r(x)$



b)

$y = r \circ h(x)$



$r \circ h(-1) = r(h(-1)) = r(3) \approx .75$
$r \circ h(0) = r(h(0)) = r(2) = 1$
$r \circ h(1) = r(h(1)) = r(1) = 2$
$r \circ h(2) = r(0) = \varnothing$ (no solution)
$r \circ h(3) = r(-1) = -2$
$r \circ h(4) = r(-2) = -1$
$r \circ h(5) = r(-3) \approx -.5$
$r \circ h(6) = r(-4) \approx -.3$

family:
reciprocal:
$y = \frac{1}{x}$

(20) $h(t)=\frac{1}{t}$ and $j(t)=\frac{1}{t+3}$

a) Find $h\circ j(t)$:

$$h(j(t))=\frac{1}{j(t)}=\frac{1}{\left(\frac{1}{t+3}\right)}=\frac{1}{1}\cdot\frac{t+3}{1}=t+3$$

b) Domain of $h\circ j(t)=\{t\in\mathbb{R}\mid t\neq -3\}$. Note that while the domain of $y=t+3$ is all $\mathbb{R}$, -3 is not in the domain of j.

c) F $j\circ h(t)$:

$$j(h(t))=\frac{1}{h(t)+3}=\frac{1}{\frac{1}{t}+3}=\frac{1}{\left(\frac{1}{t}+\frac{3t}{t}\right)}=\frac{1}{\left(\frac{1+3t}{t}\right)}=\frac{1}{1}\cdot\frac{t}{1+3t}=\frac{t}{1+3t}$$

d) Domain of $j\circ h(t)=\{t\in\mathbb{R}\mid t\neq -\frac{1}{3},0\}$. Note that while 0 is in the domain of $\frac{t}{1+3t}$, it is not in the domain of h.

(24)

Speed	heart rate
6 mph	121 bpm
10 mph	165 bpm

Linear relationship

a)

Find $R(x)$:

$R(x)=mx+b \rightarrow \frac{y_2-y_1}{x_2-x_1}=m \rightarrow \frac{165-121}{10-6}=\frac{44}{4}=11$

$R(x)=11x+b \rightarrow b=R(x)-11x \rightarrow 121-11(6)=6=55$

$R(x)=11x+55$

b) Resting heart rate: 55 bpm

c) Find $R^{-1}(x) \rightarrow R\circ R^{-1}(x)=x \rightarrow R(y)=x \rightarrow 11y+55=x$
$\rightarrow 11y=x-55 \rightarrow y=\frac{1}{11}x-5=R^{-1}(x)$

d) $R^{-1}(100)=\frac{45}{11}\approx 4$. Interpretation: the athlete must run a 4 mph to reach a heart rate of 100 bpm

e) R: domain: $\{0<x<15\}$ (assuming the athlete has a top speed of 15 mph: answers may vary). range: $\{55<R<220\}$
R^{-1} domain: $\{55<x<220\}$. range: $\{0<R^{-1}<15\}$

(32)

a) $p(x) = 20\%$ markdown
$r(x) = 20\%$ markup

$p(x) = .8x$
$r(x) = 1.2x$

b) $r \circ p = r(p(x)) = r(.8x) = 1.2(.8x) = .96x \neq x$, so r and p are not inverses.

c) Find $p^{-1}(x)$:

$p \circ p^{-1}(x) = x \rightarrow p(y) = x \rightarrow .8y = x \rightarrow y = \frac{x}{.8} \rightarrow y = 1.25x$

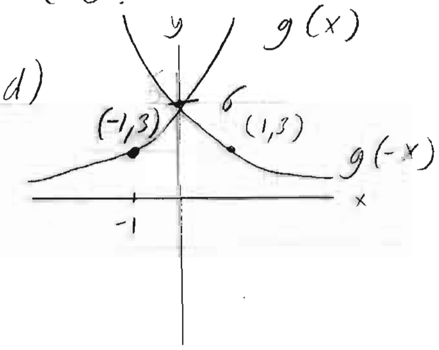
Chapter 4

(2) $g(x) = 3 \cdot 2^{x+1}$

a) $g(0) = 3 \cdot 2^{0+1} = 3 \cdot 2^1 = 3 \cdot 2 = 6$
$g(-2) = 3 \cdot 2^{-2+1} = 3 \cdot 2^{-1} = 3 \cdot \frac{1}{2} = \frac{3}{2}$
$g(3/2) = 3 \cdot 2^{\frac{3}{2}+1} = 3 \cdot 2^{\frac{5}{2}} = 3 \cdot (2^5)^{\frac{1}{2}} = 3 \cdot 32^{1/2} = 3\sqrt{32} =$
$3\sqrt{16 \cdot 2} = 3\sqrt{16} \cdot \sqrt{2} = 3 \cdot 4\sqrt{2} = 12\sqrt{2}$
$g(-\frac{1}{3}) = 3 \cdot 2^{-\frac{1}{3}+1} = 3 \cdot 2^{\frac{2}{3}} = 3 \cdot (2^2)^{\frac{1}{3}} = 3 \cdot 4^{\frac{1}{3}} = 3\sqrt[3]{4}$

b) Domain $= \mathbb{R}$, Range $= \{g > 0\}$ (Recall that if a base a is positive, there is no power y of the base for which $a^y \leq 0$.)

c), d)



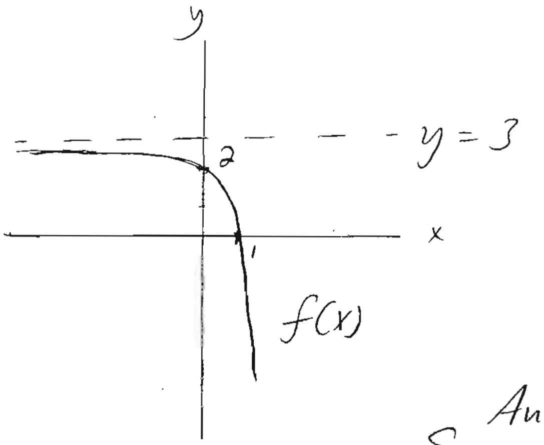
e) $g(-x) = 3 \cdot 2^{1-x}$

This is the same as $6\left(\frac{1}{2}\right)^x$ because

$3 \cdot 2^{1-x} = 3 \cdot \frac{2^1}{2^x}$ ← (law 2)

$= 6 \cdot \frac{1}{2^x} = 6 \cdot \frac{1^x}{2^x} = 6 \cdot \left(\frac{1}{2}\right)^x$ ← (law 5)

⑧ Write a formula:

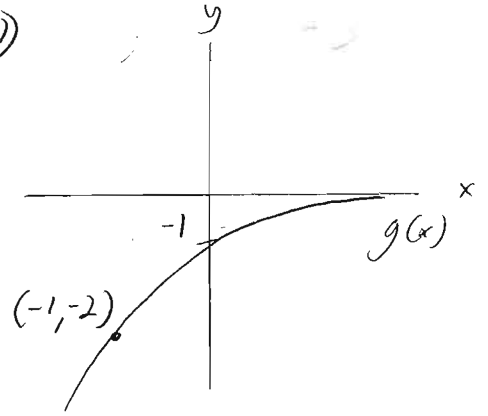


This is $y = a^x$ flipped vertically and shifted up by 3. So $f(x) = -ca^x + 3$

Now $f(0) = 2$. That is,
$-ca^0 + 3 = 2 \rightarrow -c + 3 = 2, \rightarrow c = 1$
giving $f(x) = -a^x + 3$

And $f(1) = 0: -a + 3 = 0 \rightarrow a = 3$.
So we have: $f(x) = -3^x + 3$

⑩



This is a flipped exponential but there is no vertical shift, so we start with $g(x) = -ca^x$.

$g(0) = -1 \rightarrow -c = -1 \rightarrow c = 1$, so
$g(x) = -a^x$

$g(-1) = -2 \rightarrow -a^{(-1)} = -2 \rightarrow \frac{1}{a} = 2 \rightarrow a = \frac{1}{2}$.

Giving us: $g(x) = -\left(\frac{1}{2}\right)^x$

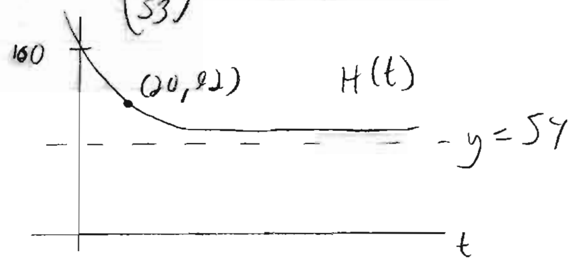
⑬

seconds after dash	heart rate
0 s	160 bpm
20 s	92 bpm
∞	$\geq$ 54 bpm

This is a decreasing exponential function shifted up by 54 units:

$H(t) = ca^t + 54$

$H(0) = 160 = ca^0 + 54 \rightarrow c = 106.\quad H(t) = 106a^t + 54$
$H(20) = 92 = 106a^{20} + 54 \rightarrow 106a^{20} = 38 \rightarrow a^{20} = \frac{19}{53}$
$\rightarrow (a^{20})^{\frac{1}{20}} = \left(\frac{19}{53}\right)^{\frac{1}{20}} \rightarrow a \approx .95$. So $\underline{H(t) = 106 \cdot .95^t + 54}$



(14)

days learning	words learned
0	20
5	100
∞	500

This is an upside-down exponential function with a horizontal asymptote at $y=500$. So $ESP(t) = -ca^x + 500$

$ESP(0) = 20 = ca^0 + 500 \rightarrow c = 480$. So $ESP(t) = -480a^x + 500$

$ESP(5) = 100 = -480a^5 + 500 \rightarrow 480a^5 = 400 \rightarrow a^5 = \frac{5}{6}$

$\rightarrow (a^5)^{1/5} = \left(\frac{5}{6}\right)^{1/5} \rightarrow a \approx .9642$. So $\underline{ESP(t) = -480(.9642)^t + 500}$