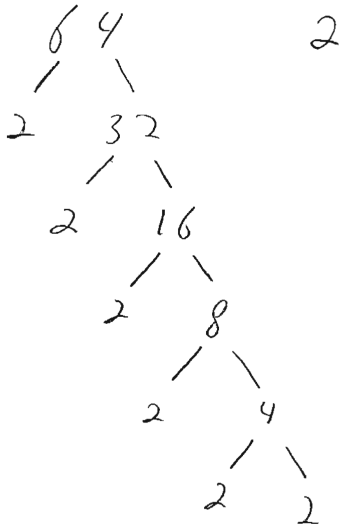
Precalc Solutions – Week 9

Ch 4

(23) Evaluate each without a calculator:

a) $\log_2(64)$

Question: "to what power must we raise 2 to get 64? The prime factorization of 64 is:



$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6$. So $\log_2(64) = 6$.

b) $\log_{16}(64)$

$16^2 = 256$, so there is no integer power of 16 equal to 64. Instead, find the factors of 16 and 64:
We know $64 = 2^6$ and $16 = 2^4$
Rewrite $\log_{16}(64) \to \log_{2^4}(2^6)$ or $(2^4)^x = 2^6$.

$2^{4x} = 2^6 \to 4x = 6 \to x = 6/4 = 3/2$. So $\log_{16}(64) = 3/2$

c) $\log_3\left(\frac{1}{27}\right) \rightarrow 3^x = \frac{1}{27}$

using rules of logs: $\log_3\left(\frac{1}{27}\right) = \log_3(1) - \log_3(27)$

$3^0 = 1$ and $3^3 = 27$, so $\log_3\left(\frac{1}{27}\right) = 0 - 3 = -3$

d) $\log_{27}(3)$ or $27^x = 3$

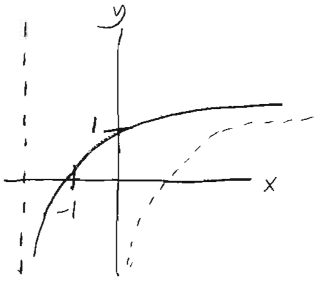
Notice that $27 = 3^3$, so we have $(3^3)^x = 3$
$3^{3x} = 3^1$, $3x = 1$, $x = \frac{1}{3}$. So $\log_{27}(3) = \frac{1}{3}$

e) $\ln(e^{3.45})$

log identities: $\log_a(a^x) = x$. $\ln = \log_e$. So

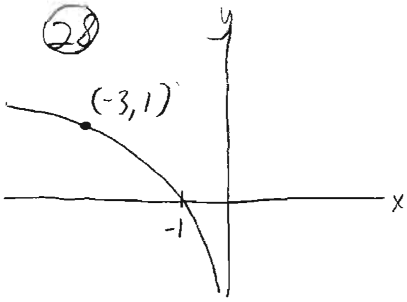
$\ln(e^{3.45}) = \log_e(e^{3.45}) = 3.45$.

(27)



This log function $\log_a x$ has been shifted 2 units to the left. It becomes $\log_a(x+2)$. To find the base, notice that $\log_a(a) = 1$. If this graph is shifted back to its original position, the value $\log_a(x) = 1$ appears at $x = 2$. So 2 is the base. The function is thus $\log_2(x+2)$

(28)



Original: $\log_a x$
Horizontal flip: $\log_a x \rightarrow \log_a(-x)$.
$\log_a(-3) = 1$, so $a = 3 \rightarrow \log_3(-x) = f(x)$

(34) Given $g(x) = \ln(3x-2)$, find $g^{-1}(x)$

$y = \ln(3x-2) \rightarrow x = \ln(3y-2) \rightarrow e^x = e^{\ln(3y-2)}$
$\rightarrow e^x = 3y - 2 \rightarrow 3y = e^x + 2 \rightarrow \boxed{y = \frac{1}{3}e^x + \frac{2}{3}}$

(Review rules of logs & exponents for each step)

(41) Rewrite each expression as a single log with coefficient 1:

a) $2\log_{10}(x) + \frac{1}{2}\log_{10}(3x)$

$\log_{10}(x^2) + \log_{10}[(3x)^{1/2}]$

$\log_{10}(x^2 \cdot (3x)^{1/2})$

$\log_{10}(\sqrt{3}\, x^2 \cdot x^{1/2}) \longrightarrow \log_{10}(\sqrt{3}\, x^{5/2})$

b) $\ln(2t) - 4\ln\left(\frac{1}{t}\right)$

$\ln(2t) - \ln\left[\left(\frac{1}{t}\right)^4\right]$

$\ln(2t) - \ln\left(\frac{1}{t^4}\right)$

$\ln(2t) - \ln(t^{-4})$

$\ln\left(\frac{2t}{t^{-4}}\right)$

$\ln\left(\frac{2t^1}{t^{-4}}\right)$

$\ln(2t^{1-(-4)}) \longrightarrow \ln(2t^5)$

c) $\ln(400) + .95t\ln(10)$

$\ln(400) + \ln(10^{.95t})$

$\ln(400 \cdot 10^{.95t})$

(44) Solve exactly, then calculate an approximation to 3 dec. places.

a) $2^x = .96 \rightarrow \ln(2^x) = \ln .96 \rightarrow x \ln 2 = \ln(.96)$
$\rightarrow x = [\ln(.96)]/[\ln(2)] \approx -.059$

b) $.96^x = 2 \rightarrow \ln(.96^x) = \ln(2) \rightarrow x \ln(.96) = \ln(2)$
$\rightarrow x = \dfrac{\ln(2)}{\ln(.96)} \approx -16.98$.

c) $2 \cdot 10^{3x+1} = 85$
$\ln(2 \cdot 10^{3x+1}) = \ln(85)$
$\ln 2 + \ln(10^{3x} \cdot 10) = \ln(85)$
$\ln 2 + \ln(10^{3x}) + \ln 10 = \ln 85$
$\ln 2 + 3x \ln 10 + \ln 10 = \ln 85$
$3x \ln 10 + \ln 20 = \ln 85$
$3x \ln 10 = \ln 85 - \ln 20$
$3x \ln 10 = \ln\left(\frac{85}{20}\right)$
$x = \frac{1}{3}\left[\ln\left(\frac{85}{20}\right)\right]/\ln 10 \approx .209$

(46)

	Country A	Country B
Starting pop.	5.24 mil	3.16 mill
yearly growth rate	1.3%	2.1%

Equation: $A = 5.24(1.013)^x$ $B = 3.16(1.021)^x$

Set $A = B$ and solve for x:

$5.24(1.013)^x = 3.16(1.021)^x$

$5.24/3.16\,(1.013)^x = (1.021)^x \rightarrow \dfrac{5.24}{3.16} = \dfrac{(1.021)^x}{(1.013)^x}$
$\rightarrow \dfrac{5.24}{3.16} = \left(\dfrac{1.021}{1.013}\right)^x \rightarrow \ln\left(\dfrac{5.24}{3.16}\right) = \ln\left[\left(\dfrac{1.021}{1.013}\right)^x\right]$

$x \ln\left(\dfrac{1.021}{1.013}\right) = \ln\left(\dfrac{5.24}{3.16}\right) \rightarrow x = \dfrac{\ln\left(\frac{5.24}{3.16}\right)}{\ln\left(\frac{1.021}{1.013}\right)} \approx 64.29$ years.

(52) Show that $M(x) = \log_{10}\left(\frac{x}{.001}\right)$ is equivalent to $M(x) = \log_{10}(x) + 3$

$$\log_{10}\left(\frac{x}{.001}\right) \stackrel{?}{=} \log_{10}(x) + 3$$

Simplify: $\log_{10}(1000x) \stackrel{?}{=} \log_{10}(x) + 3$

$$10^{\log_{10}(1000x)} \stackrel{?}{=} 10^{\log_{10}(x) + 3}$$

log identity: $1000x \stackrel{?}{=} x \cdot 10^3 \rightarrow 1000x = 1000x$ ✓

there is no inconsistency. Another way to show these 2 equations are equivalent is by turning one into the other by changing the form algebraically. We will try to "manipulate $\log_{10}\left(\frac{x}{.001}\right)$ to show it is the same as $\log_{10}(x) + 3$:

$$\log_{10}\left(\frac{x}{.001}\right) \rightarrow \log_{10}(1000x)$$

$\rightarrow \log_{10}(1000) + \log_{10}(x)$ law 2

$3 + \log_{10}(x)$ ✓ thus they are equivalent

(55) A postmenopausal woman loses 5% of bone mineral content each year (decay rate of 5%). The equation is $B = .95^x$. After how many years will she have lost $\frac{1}{2}$ of her mineral bone matter? (She will have $\frac{1}{2}$ left)

$$.5 = .95^x \rightarrow \log .5 = \log(.95^x)$$

$$\rightarrow \log(.5) = x \log(.95) \rightarrow x = \frac{\log .5}{\log .95} \approx 13.5 \text{ years.}$$

(59) $\ln y = 3x + \ln 2$

express y as a function of x without logs.

$\ln y = 3x + \ln 2$

$e^{\ln y} = e^{3x + \ln 2}$

$y = e^{3x} \cdot e^{\ln 2}$

$y = e^{3x} \cdot 2$, $\boxed{y = 2e^{3x}}$

(60) $\ln y = 3\ln x + \ln 2$

$e^{\ln y} = e^{3\ln x + \ln 2}$

$e^{\ln y} = e^{3\ln x} \cdot e^{\ln 2}$

$e^{\ln y} = (e^{\ln x})^3 \cdot e^{\ln 2}$

$y = x^3 \cdot 2$, $\boxed{y = 2x^3}$