Introduction to Natural Science Take Home Exam

Name:

- This test forms part of our assessment of your personal learning in this program. Your work must be done independently of others.
- Attempt all questions on this test. Do not leave answers blank. Marks will be given for partial answers, so show all your working.
- This is an open book test. You may consult your textbooks, notes, homework and workshop solutions.
- Your completed test is due at 10:30 am on Monday Dec 3rd.

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1. Each of the following statements suggests a relationship between two variables. Identify the *independent* and *dependent* variables and indicate whether a *linear function, exponential function or power function* best describes the suggested relationship. Explain your reasoning.

   (a) The height of a healthy Douglas fir tree increases at a rate of about 4 ft per year.

   (b) When a Douglas fir tree doubles it height the amount of wood it contains quadruples.

   (c) The number of old growth Douglas fir trees in the Olympic rain forest has been decreasing by 5% every year for the last decade.

2. For each of the following functions decide whether $f(x)$ is a linear function, an exponential function, a power function or none of these.

   (a) $f(x) = 5x^6$

   (b) $f(x) = 2x - 1$

   (c) $f(x) = 3x^2 + 2x$

   (d) $f(x) = 2(3)^x$

3. Solve the following equations

   (a) $5^x + 2 = 9$

   (b) $\log x = 2 \log 3$

4. The blood alcohol content (BAC) is a measure of the how much alcohol is in the blood stream. Suppose the BAC for an individual decays exponentially from 0.10 to 0.04 in 3 hours.

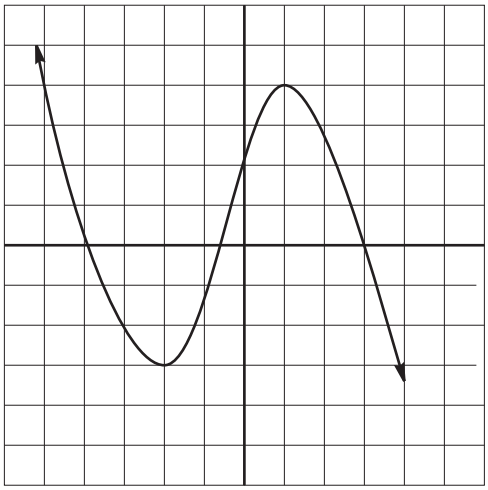
   (a) Write down an expression describing the BAC as a function of time in the form $y = Ce^{kx}$ assuming the initial BAC is 0.10.

   (b) What is the BAC after 2 hours?

   (c) How long will it take for the BAC to fall below 0.01?

5. The graph of a function $y = f(x)$ is plotted below. On the same set of axis sketch the graph of the transformed functions below, making sure to label each graph.

$$y = -f(x+1)\,, \qquad \text{and} \qquad y = f(2x) + 1$$



6. Given the functions

$$f(x) = 10x^3 \qquad g(x) = 9(10^x) - 1 \qquad h(x) = 2\log(x)$$

Find and simplify as far as possible

(a) $f^{-1}(x)$

(b) $g^{-1}(x)$

(c) $h \circ f(x)$

(d) $g \circ h(x)$

7. The number of frogs is in tropical regions is on the decline due in part to global warming. Suppose that the number of frogs in a certain habitat in 1980 was 2400 and that in 1990 that number had decreased to 1800.

   (a) Find the average rate of change in frogs per year during the 80's

   (b) Find the percentage change in frogs during the 80's

   (c) Assuming a linear decline find the population of frogs as a function of time (measured in years since 1980).

   (d) Assuming an exponential decline find the population as a function of time (measured in years since 1980).

   (e) The frogs will become extinct if the population falls to 2 frogs. How long will this take according to each of the two models?

8. The speed at which a ball hits the ground v is directly proportional to the square root of height h it is dropped from.

   (a) Express this relationship as a power function in the form $v = kh^p$

   (b) Find the value of k, with units, if the speed is 6.0 m/s when the ball is dropped from 2.0 m.

   (c) If the ball is dropped from 8.0 m what is the new speed?

   (d) If the ball lands with speed 10 m/s from what height was it dropped?

9. It is a little known fact that the inhabitants of Mercury used a temperature scale based on the melting and boiling points of the element mercury (-40 ^{o}C and 357 ^{o}C respectively). On their temperature scale the melting point of mercury is called 0 ^{o}M and the boiling point of mercury is called 100 ^{o}M. Find a linear relationship that will convert temperature in ^{o}M to temperature in ^{o}C and use that relationship to determine what a Mercurian body temperature of 30 ^{o}M is in Celsius.