

Complete all the questions in the space provided showing all your work. You have 90 minutes to complete this test.

1. For which of the following relations is the variable y a function of the variable x ? For which is x a function of y . Explain your answer.

Hint: For a function there should be a unique output for each input. If you can think of a case where one input could have more than one output it is not a function.

- (a) x is the pulse rate and y is the body temperature of a patient as measured at various times over the day.
- (b) Given squares of different size let x be the perimeter of the square y be the area of the square.
- (c) x is the name of a town y is the name of a country that it is in.

2. Functions $f(x)$, $g(x)$ and $h(x)$ are defined as follows:

$$f(x) = 1/x^2, \quad g(x) = 3 + \sqrt{x}, \quad h(x) = 2x^2 - 3x$$

- (a) What are the domain and range of $f(x)$ and $g(x)$.
- (b) Evaluate $g(4)$ and $h(-1)$.
- (c) Find and simplify expressions for $h(2x)$ and $f(x/2)$

3. The following two statements were taken from an article on wireless communication
- (a) According to industry sources the global wireless infrastructure market is growing at 20% annually.
 - (b) In the US an average of 26 more people sign up for wireless phone service each minute.

Which of these two statements implies linear growth and which exponential growth? Explain your answer and also explain how they could both be true?

Hint: We are assuming that these measures of change are constant.

4. The following formulas give the populations (in 1000s) of three different cities, A , B and C , as a function of time since 1980. $P_A = 400 + 9t$, $P_B = 270(1.021)^t$ and $P_C = 600(0.978)^t$.

- (a) Describe in words how each of these populations is changing over time. In your description make sure you indicate the physical meaning of each number in the formula and give units when relevant.

Remember the difference between growth factor and percentage change. How do you find the percentage change from the growth factor?

- (b) If the growth continues as described by these formulae which city will eventually be the largest?

Don't just plug in values. Sketch a graph of how these populations change.

5. If a population of rabbits doubles in size every 7 years by what factor does the population grow in one year? What is the annual growth rate?

What is the growth factor for 7 years? How do you change a 7 year growth factor to a 1 year growth factor?

6. In 1965 the atmospheric CO₂ concentration was 320 ppm and in 1995 it was 360 ppm. This question is about measure of change. It is crucial you understand the difference.
- (a) What was the average rate of change in CO₂ concentration per year between 1965 and 1995?
Hint: The average rate of change is the slope in ppm/year
- (b) What was the percentage change in CO₂ concentration between 1965 and 1995? Hint: This is the ratio of the change in CO₂ over the initial amount, expressed as a percentage.
- (c) By what factor did the concentration increase in that time? Hint: A factor is what you multiply by to get from initial to final values – in this case from 320 to 360.
- (d) Assuming CO₂ concentration y grows linearly with years t since 1965 find an expression for y as a function of t .
- (e) Assuming CO₂ concentration y grows exponentially with years t since 1965 find an expression for y as a function of t . Hint: make sure you change the growth factor from a 30 year period to a 1 year period
- (f) Use both models to predict the CO₂ concentration in 2050.