Introduction to Natural Science Probability Worksheet 1 Solutions

We often consider outcomes of events that are combinations of two simpler events. For example, getting a double six when throwing two dice is a combination of getting a six on the first dice and a six on the second. The probability of combined events follow special rules when certain conditions apply. Here are some special cases.

Independent Events: Two events are independent if the occurrence of one does no affect the probability of the other occurring. (e.g. Getting a heads on one coin will not affect the probability of getting heads on the second.)

Mutually Exclusive Events: Two events are mutually exclusive if the occurrence of one means the other cannot occur. (e.g. When tossing a die, the events A="getting a six" and B="getting a 1" are mutually exclusive.)

Exhaustive Events: Two events are exhaustive if whenever one of the events doesn't occur the other one must. (e.g. When we toss a coin heads and tails are exhaustive events).

1. Decide whether or not the following events are independent, mutually exclusive, and/or exhaustive.

   (a) The weather is nice; I walk to work.
   If the weather is nice I am more likely to walk to work, so they are not independent. It is possible that I will walk to work when it is nice so they are not mutually exclusive. If it is not nice it is possible that I will not work so they are not exhaustive.

   (b) I cut an Ace; you cut a King.
   These events are independent (one does not affect the other), not mutually exclusive (they can both happen) and not mutually exclusive (there are other cards to draw).

   (c) Mrs Smith has cold; Mr Smith has a cold.
   These events are not independent – if the wife has a cold the husband is more likely to. They are not mutually exclusive – they can both have a cold at the same time. They are not mutually exclusive – its possible for neither to have a cold.

   (d) Mrs Smith has a tooth ache, Mr Smith has tooth ache.
   Similar to above except they are independent events – Mrs Smith having a tooth ache won't make Mr Smith more likely to have one.

   (e) I get a number less than 4 on a die toss; I get a number greater than 2 on the same die toss.
   Knowing that the number on a dice toss is less than 4 changes the probability that the number is greater than 2 (it becomes less likely) so the events are not independent. They are not mutually exclusive since I could get a 3 which is both less than 4 and greater than 2. They are exhaustive – all scores on a die are either less than 4 or greater than 2.

   (f) The base at a DNA site is adenine, The base at the same site is a pyrimidine.
   They are not independent. If I get an adenine I know with certainty that it is not a pyrimidine. They are mutually exclusive – an adenine is not a pyrimidine. They are not exhaustive – guanine is not a pyrimidine and also is not adenine.

2. Can two events be mutually exclusive and independent? If so, given an example. If not explain why not.
   No. If two events are mutually exclusive then knowing that one occurs means you know with certainty that the other hasn't.

3. If two events are mutually exclusive are they also exhaustive? If so explain, if not give a counter example.
   Not necessarily. For example, getting a 1 and getting a 3 on a dice are mutually exclusive, but not exhaustive because you could also get a 2,4,5,6.

4. If two events are exhaustive, are they also mutually exclusive? If so explain, if not give a counter example.
   Not necessarily. For example in a family of 3 the events "getting least one son" and "getting at least one daughter" are exhaustive events because no matter how many of each gender you get you either have at least one son or at least one daughter. They are not mutually exclusive since, for example, having a one son and two daughters satisfies both events.

5. If two events, A and B are independent then $P(A \text{ and } B) = P(A)P(B)$. This is called the law of multiplication. For example, when tossing two dice the probability of getting a six on the first dice and six on the second is $(\frac{1}{6})(\frac{1}{6}) = \frac{1}{36}$. Use this law to answer the following questions:

   (a) Two dice are thrown, find the probability that the first is even and the second shows a three.
   $P(\text{even}) = \frac{1}{2}$ and $P(3) = \frac{1}{6}$.
   Since they are independent then $P(\text{even on first and 3 on second}) = (\frac{1}{2})(\frac{1}{6}) = \frac{1}{12}$.

   (b) Two sisters each have two children. What is the probability that they both have two girls?
   The probability of a two children family having two girls is $(\frac{1}{2})(\frac{1}{2}) = \frac{1}{4}$. So the probability of the first sister having two girls is $\frac{1}{4}$ and the probability of the second girl having two girls is $\frac{1}{4}$, so the probability of both having two girls is $(\frac{1}{4})(\frac{1}{4}) = \frac{1}{16}$.

6. When two events, A and B are mutually exclusive what is the value of $P(A \text{ and } B)$? Using this answer and the general law of probability addition express $P(A \text{ or } B)$ as a combination of the separate probabilities $P(A)$ and $P(B)$. Explain why using a Venn diagram.
   If two events are mutually exclusive they both cannot happen at the same time so $P(A \text{ and } B) = 0$. Now since in general $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, it follows that $P(A \text{ or } B) = P(A) + P(B)$ for mutually exclusive events. The Venn diagram has two non-intersecting circles.

7. If two events, A and B are exhaustive what is the value of $P(A \text{ or } B)$? If they also mutually exclusive what is the value of $P(A) + P(B)$?
   For exhaustive events one or both must happen, so $P(A \text{ or } B) = 1$. Now since $P(A \text{ or } B) = P(A) + P(B)$ for mutually exclusive events it follows that $P(A) + P(B) = 1$ for mutually exclusive and exhaustive events.

8. Given an event A, the event "not A" (the event that A does nor occur) is denoted A'. Explain why $P(A') = 1 - P(A)$ using the your answer to the question above above. This is the law for "not A". Often it is easier to calculate the probability for "not A" then the probability for A.
   A and A' are mutually exclusive and exhaustive events, so $P(A) + P(A') = 1$. Therefore it follows that $P(A') = 1 - P(A)$.

9. Use the laws above to answer the following questions

   (a) In a family of 4 children, what is the probability of getting only girls?
   The probability that any given child is a girl is $\frac{1}{2}$, so the probability that the first, second, third and forth is a girl is $(\frac{1}{2})(\frac{1}{2})(\frac{1}{2})(\frac{1}{2}) = \frac{1}{16}$.

   (b) In a family of 4 children, what is the probability of getting at least one boy?
   If A is the event of getting at least one boy then A' is the event of not getting at least one boy – that is getting all girls. $P(A) = 1 - P(A') = 1 - \frac{1}{16} = \frac{15}{16}$.

   (c) Toss a coin and draw a card, what is the probability of getting a head and an ace?
   $P(\text{Head}) = \frac{1}{2}$ and $P(\text{Ace}) = \frac{1}{13}$ so since they are independent events $P(\text{Head and Ace}) = (\frac{1}{2})(\frac{1}{13}) = \frac{1}{26}$.

   (d) What is the probability of getting a head or an ace? (Hint: You will need to use the more general law of addition).
   Since these ar not mutually exclusive events we use the general law $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = \frac{1}{2} + \frac{1}{13} - \frac{1}{26} = \frac{13+2-1}{26} = \frac{14}{26} = \frac{7}{13}$.

   (e) Suppose that if a father is blue eyed and a mother is brown eyed then the probability of having a child with blue eyes is $\frac{1}{4}$. If such parents have four children what is the probability of getting at least one blue eyed child.
   Let A be the event of getting at least one blue eyed child. Then A' is the probability of getting no blue eyed child. $P(A') = (\frac{1}{4})^4 = \frac{1}{256}$ so $P(A) = 1 - P(A') = 1 - \frac{1}{256} = \frac{255}{256}$.

10. If you are finished these questions come to me and ask for a Fermi Question.