1. The following table shows the results of a survey of 60 students showing the relation between hair color and gender.

	Blond	Brown	Red	Total
Male	9	20	1	30
Female	16	12	2	30
Total	25	32	3	60

What is the probability that a randomly selected student

(a) is blond?
P(blond)= $\frac{25}{60} = \frac{5}{12}$.

(b) is a blond haired male?
P(blond and male)= $\frac{9}{60} = \frac{3}{20}$.

(c) is blond given that he is known to be male?
P(blond | male)= $\frac{9}{30} = \frac{3}{10}$.

(d) is male given that the person is known to be blond?
P(male | blond)= $\frac{9}{25}$.

2. Two bags are given. The first bag contains 4 white and 2 black beads. The second bag contains 3 white and 3 black beads. One of the bags is chosen at random and a bead is drawn from that bag.

(a) What is the probability that the bead is white?
$P(\text{white}) = P(\text{bag 1 and white}) + P(\text{bag 2 and white}) = (\frac{1}{2})(\frac{4}{6}) + (\frac{1}{2})(\frac{3}{6}) = \frac{7}{12}$.

(b) Given that the bead is white what is the probability that it came from the first bag?
$P(\text{bag 1} \mid \text{white}) = \frac{P(\text{white and bag 1})}{P(\text{white})} = \frac{(\frac{1}{2})(\frac{4}{6})}{\frac{7}{12}} = \frac{4}{7}$.
If you got this answer by reasoning that there are 7 white stones in total and 4 in the white bag then this is also correct reasoning.

3. The allele for color blindness is carried on the X chromosome and is recessive. A female will only be color blind if she has both alleles for color blindness, but a male will be color blind if he has one allele for color blindness.

   (a) Suppose a women who is heterozygous for the color blindness gene has a child with a man who is not color blind. What is the probability her son will be color blind?
   Since he is a boy he has a Y from his dad. To be color blind he would have to get the color-blind X from his mom, which occurs with probability $\frac{1}{2}$.

   (b) Suppose the women and man have a daughter. If the daughter has a son with a man who is not color blind what is the probability that the son is color-blind?
   The daughter get the color-blind gene from her mom with probability $\frac{1}{2}$ and passes it on to her son with probability $\frac{1}{2}$. The probability that both happen is $(\frac{1}{2})(\frac{1}{2}) = \frac{1}{4}$.

   (c) If the daughter's son turns out not to be color-blind, what is the probability that she does carry the color blind gene?
   This is a bit harder. To find the conditional probability that she carries the gene given that her son doesn't we need to take the probability that her son is not color blind and she carries the gene (which is $(\frac{1}{2})(\frac{1}{2})$ based on a similar argument to (b)) and divide it by the probability that her son is not color blind. (which is $1 - \frac{1}{4} = \frac{3}{4}$ from (b)). Hence the probability is $\frac{1}{4}/\frac{3}{4} = \frac{1}{3}$.

4. A lab wants to stain cells for an experiment. Young cells stain with probability 90% where as old cells stain with probability 50%.

   (a) If 30% of cells are young what fraction of the cells are expected to stain properly?
   This a combination of getting a young stained cell or an old stained cell, the probability for which is
   $(0.3)(0.9) + (0.7)(0.5) = .62$ or 62%.

   (b) Suppose the experimenter selects a stained cell, what is the probability it is young?
   This is the conditional probability
   $$\text{P(young} \mid \text{stained)} = \frac{\text{P(young and stained)}}{\text{P(stained)}} = \frac{(0.3)(0.9)}{0.62} = \frac{27}{62} = 0.435 \text{ or } 43.5\%.$$

   (c) Does staining increase the probability of finding young cells compared with not staining? Is so by how much?
   It increases the probability from 30% to 43.5%. This is a small but perhaps helpful increase.

5. A test for a disease shows a positive result in 95% of all cases when the disease is actually present and 1% of cases when it is not. Suppose 1 out of very 1000 people has this disease.

   (a) What is the probability that a randomly chosen person tests positive?
   To test positive you either have the disease and get a true-positive or don't have the disease and get a false-positive.
   This occurs with probability $(\frac{1}{1000})(\frac{95}{100}) + (\frac{999}{1000})(\frac{1}{100}) = \frac{1094}{100000}$.

   (b) If a person tests positive for the disease, what is the probability that she has it?
   This is a conditional probability:
   $$\text{P(has disease} \mid \text{tests positive)} = \frac{\text{P(has disease and tests positive)}}{\text{P(tests positive)}} = \frac{(\frac{1}{1000})(\frac{95}{100})}{\frac{1094}{100000}} = \frac{95}{1094} =$$
   0.087 or 8.7%.

   (c) If you wanted to know if you had this disease would it help to take this test? If your doctor believes, given other observations, that you are actually in a risk group that has a 25% chance of having the disease, then how confident will she be that you have the disease if your test is positive? Discuss the implications of this for doctors who wish to diagnose an illness.
   Since there is only a 8.7% chance that a person who tests positive has the disease this test will not help you confirm that you have it. If other factors indicate you have a 25% chance of having the disease, the positive test result will raise that probability to $\frac{(\frac{1}{4})(\frac{95}{100})}{(\frac{1}{4})(\frac{95}{100})+(\frac{3}{4})(\frac{1}{100})} = \frac{95}{98} = 0.97$ or 97%. This means that a doctor most rely on more than a test to make a diagnosis (unless it is a lot more sensitive than this one.).

6. Suppose you have a batch of purple flowering pea plants, 60% of which are genotype Pp and 40% are genotype PP. If these plants are used to cross with a white flowering plant, what fraction of the offspring will produce purple flowers?
If a Pp plant is selected the probability the offspring is purple is $\frac{3}{4}$. If a PP plant is select the offspring will certainly be purple. Therefore the probability of getting a purple plant is $(0.6)(\frac{3}{4}) + (0.4)(1) = .85$ or 85%.

7. If you are finished these questions come to me and ask for a Fermi Question.