1. For each of the situations given below state, with reasons, whether or not the random variable X has a binomial distribution . In the cases where it is binomial find the expectation value of X, $\mu = E[X]$. Also find the probability that X equals that value. That is, find $P(X = \mu)$.

   (a) 5% of the population is color blind. If 40 people are chosen at random X is the number who are color blind.
   This is a binomial distribution, with 40 trials, probability of success is 0.05 and expectation value is $\mu = (0.05)(40) = 2$. For this, $P(X=2) = \binom{40}{2}(0.95)^{38}(0.05)^2 = 0.27$.

   (b) A pencil case contains 4 pens and 5 pencils. If two items are removed without replacement X is the number of these that are pencils.
   This is not binomial because the probability of the second trial is dependent on the first.

   (c) 35 people are selected at random. X is the number of people with birthdays on a Friday.
   This is binomial. The probability of success is $\frac{1}{7}$, the number of trials is 35 and the expected value of X is $(35)(\frac{1}{7}) = 5$. For this $P(X=5) = \binom{35}{5}(\frac{1}{7})^5(\frac{6}{7})^{30} = 0.189$.

   (d) A coin is tossed until a head appears, Let X be the number of times it is tossed before a head appears.
   This is not binomial. The number of trials is not fixed, but depends on the outcome of the previous toss.

2. A women who carries one allele for color blindness has 5 children with a man who is not. Find the probability distribution for X, the number of color blind children she has. What is the expected number of color blind children?
   This is a binomial distribution with $n = 5$ and $p = \frac{1}{4}$ (based on the genetic model discussed in biology). So we find the probabilities using $P(x) = \binom{n}{x}(\frac{1}{4})^x(\frac{3}{4})^{n-x}$. The expectation value is $\mu = np = 5\frac{1}{4} = 1.25$

x	$P(x)$
0	$\binom{5}{0}(\frac{3}{4})^5 = \frac{243}{1024}$
1	$\binom{5}{1}(\frac{1}{4})^1(\frac{3}{4})^4 = \frac{405}{1024}$
2	$\binom{5}{2}(\frac{1}{4})^2(\frac{3}{4})^3 = \frac{270}{1024}$
3	$\binom{5}{3}(\frac{1}{4})^3(\frac{3}{4})^2 = \frac{90}{1024}$
4	$\binom{5}{4}(\frac{1}{4})^4(\frac{3}{4})^1 = \frac{15}{1024}$
5	$\binom{5}{5}(\frac{1}{4})^5 = \frac{1}{1024}$

3. Approximately 99% of carbon atoms are ^{12}C and 1% are ^{13}C. In a benzene ring of 6 carbon atoms what is the probability that:

   (a) All 6 are ^{12}C. $\binom{6}{0}(0.99)^6 = 0.941$

   (b) Exactly 5 are ^{12}C. $\binom{6}{1}(0.99)^5(0.01) = 0.0571$

   (c) Exactly 4 are ^{12}C. $\binom{6}{2}(0.99)^4(0.01)^2 = 0.0014$

   (d) If 100 benzene (C_6H_6) rings were observed in a mass spectrometer, how many would you expect to have mass 78 amu? How many would have mass 79 amu? What is the ratio?
   The benzene ring with only ^{12}C is 78 amu. We expect about 100(0.941)=94 or these. The benzene ring with one ^{13}C is 79 amu. We expect about 100(0.057)=6 of these. The ratio is about 16 to one.

   (e) Suppose benzene had a different structure, say with 8 carbon atoms, what would you expect to get for the ratio of rings with only ^{12}C to rings with one ^{13}C ?
   A similar calculation will give a ratio of about 92:8 or 11.5 to one. It will be possible to pick up this kind of difference in mass spectrometer data.

   (f) Suppose, instead, that God had given us an equal distribution of ^{12}C and ^{13}C. What do you think the mass of the most common benzene ring would be?
   We would expect the most common benzene ring to have half of each isotope and we would therefore have a peak at 81 amu. You should be able to calculate the relative heights of the peaks at 78,79,80,81,82,83 and 84 also.

4. There are 5 eggs in a nest, 3 of which are fertilized. Eggs are removed one by one and tested until the two infertile eggs have been found and removed. We define the random variable $X \in \{2, 3, 4, 5\}$ to be the number of eggs that must be removed. Find the probability distribution of X. What is the expectation value of X.
   Note: This is not a binomial distribution. If we let F denote fertile and I infertile then for $X = 2$ we must get both the infertile eggs in the first two picks – ie II. For $X = 3$ we get FII or IFI, each of which have the same probability. Similarly for $X = 4$ we have the following possibilities: $FFII$, $FIFI$, or $IFFI$ and for $X = 5$ we can have $FFFII$, $FFIFI$, $FIFFI$ or $IFFFI$. The probabilities are in the second column below.

x	$P(x)$	$xP(x)$
2	$(\frac{2}{5})(\frac{1}{4}) = \frac{1}{10}$	$\frac{1}{5}$
3	$2(\frac{3}{5})(\frac{2}{4})(\frac{1}{3}) = \frac{1}{5}$	$\frac{3}{5}$
4	$3(\frac{3}{5})(\frac{2}{4})(\frac{2}{3})(\frac{1}{2}) = \frac{3}{10}$	$\frac{6}{5}$
5	$4(\frac{3}{5})(\frac{2}{4})(\frac{1}{3})(\frac{2}{2})(\frac{1}{1}) = \frac{2}{5}$	$\frac{10}{5}$
Total	1	$\frac{20}{5} = 4$

   The expectation value of X is 4. This means we expect on average to remove 4 eggs before all the infertile eggs are gone.