1. Evaluate the following without the use of a calculator

   (a) $125^{\frac{1}{3}}$

   $125^{\frac{1}{3}} = 5$ since $125 = 5^3$

   (b) $\left(\frac{100}{9}\right)^{\frac{3}{2}}$

   $$\left(\frac{100}{9}\right)^{\frac{3}{2}} = \left(\frac{100^{\frac{1}{2}}}{9^{\frac{1}{2}}}\right)^3 = \frac{10}{3}^3 = \frac{1000}{27}$$

   (c) $\left(-\frac{1}{7}\right)^{-2}$

   $$\left(-\frac{1}{7}\right)^{-2} = \left(-\frac{7}{1}\right)^2 = (-7)^2 = 49$$

   (d) $16^{-\frac{3}{2}}$

   $16^{-\frac{3}{2}} = (16^{\frac{1}{2}})^{-3} = (4)^{-3} = \frac{1}{4^3} = \frac{1}{64}$

2. Complete the Just Algebra Exploration 2.7 question 2(b)(c)(d)(e) on pg 97 in your textbook.

3. (a) In the 1970's inflation in North America averaged 7.8% per year. What was the percentage change in prices over the whole decade?

   The yearly growth factor is 1.078 so the growth factor for a decade is $(1.078)^{10} = 2.12$ and hence the percentage change is $(2.12 - 1) \times 100\% = 112\%$.

   (b) Prices in North America rose by 32% between 1990 and 2000. What was the yearly inflation rate?

   The growth factor for the decade is 1.32 so the yearly growth factor is $(1.32)^{1/10} = 1.028$ and hence the inflation rate was 2.8%.

4. A typical cup of coffee contains 100 mg of caffeine and every hour approximately 16% of the amount of caffeine in the body is metabolized and eliminated.

   (a) Let C represent the amount of caffeine in the body in mg and t represent the number of hours since a cup of coffee was consumed and eliminated. Write C as a function of t.

   The amount of caffeine is decreasing so b is a decay factor. $b = 1 + r/100 = 1 - 0.16 = 0.84$. The initial amount of caffeine is 100 mg so $a = 100$ and hence the function is $C = 100(0.84)^t$

   (b) How much caffeine is left in the body after 3 hours?

   For 3 hours we set $t = 3$ so that $C = 100(0.84)^3 = 59$ mg

   (c) Suppose an individual cannot sleep unless she has less than 50 mg of caffeine in her blood? How many hours after drinking coffee must she wait until she goes to sleep? (This time is the half life of coffee in her system.)

   We set $C = 50$ and solve for t. Hence $50 = 100(0.84)^t \Rightarrow (0.84)^t = 0.5$. Solving by trial and error gives $t \approx 4$. solving by logs gives $t = \log(0.5)/\log(0.84) = 3.98$ hours.

5. Five years after a species of turtle was introduced into a wetland a biologist did a survey and estimated a total population of 300 turtles. In a follow up survey five years later the biologist estimated the population was 450.

(a) What is percentage change in turtles during the 5 years between the two studies? What is the average rate of change in turtles per year?

The percentage change for 5 years is $\dfrac{450 - 300}{300} \times 100\% = 50\%$

The average rate of change $\dfrac{450 - 300}{5} = 30$ turtles per year.

(b) Assuming linear growth find an expression for the number of turtles as a function of years since they were introduced. What is the initial number of turtles according to this model?

$y = b + mx$ and $m = 30$. To find the initial value b substitute $x = 5$ and $y = 300$ into the equation. Hence $300 = b + 30 \times 5 \;\Rightarrow\; b = 150$. So an expression for the number of turtles as a function of years since they were introduced is

$$y = 150 + 30x\,.$$

The initial number of turtles is 150 according to this model.

(c) Assuming exponential growth what is the yearly growth rate?

Over 5 years the growth rate is 50%. So the 5 year growth factor is 1.5 and hence the one year growth factor is $1.5^{1/5} = 1.0845$. So the yearly growth rate is 8.45%.

(d) Find the estimated initial population of turtles assuming exponential growth and hence express the number of turtles as a function of years since they were introduced.

The initial population is 5 years before the first study so it is $300(1.0845)^{-5} = 200$ turtles. Hence an expression for the number of turtles as a function of years since they were introduced is.

$$y = 200(1.0845)^x\,.$$

(e) To test which model is more accurate the biologist does a further study 15 years after the turtles were introduced and estimates a total population of 700 turtles. Which model is the better fit?

The linear model predicts

$$y = 150 + 30 \times 15 = 600 \text{ turtles.}$$

The exponential model predicts

$$y = 200(1.0845)^{15} = 675 \text{ turtles.}$$

So the exponential model is a closer fit.