1. For each of the following models, find an expression for ΔP and hence or otherwise find the equilibrium point(s).

   (a) $P_{t+1} = 0.6P_t + 2$.
   Set $\Delta P = P_{t+1} - P_t$ so that $\Delta P = 0.6P + 2 - P = -0.4P + 2$. Then we solve the equation $\Delta P = 0$ to get the equilibrium point

   $$\begin{aligned} \Delta P = 0 \;\Rightarrow&\quad 0.4P = 2 \\ \Rightarrow&\quad P = 2/0.4 = 5 \end{aligned}$$

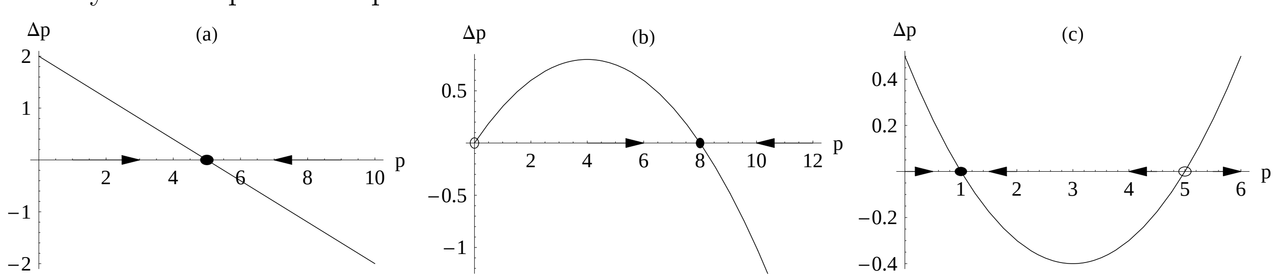
   (b) $P_{t+1} = 1.4P_t - 0.05P_t^2$.
   Set $\Delta P = P_{t+1} - P_t$ so that $\Delta P = 1.4P - 0.05P^2 - P = 0.4P - 0.05P^2$. Then we solve the equation $\Delta P = 0$ to get the equilibrium points

   $$\begin{aligned} \Delta P = 0 \;\Rightarrow&\quad 0.4P - 0.05P^2 = 0 \\ \Rightarrow&\quad P(0.4 - 0.05P) = 0 \\ \Rightarrow&\quad P = 0 \quad \text{or} \quad P = 8. \end{aligned}$$

   (c) $P_{t+1} = 0.1P_t^2 + 0.4P_t + 0.5$.
   Set $\Delta P = P_{t+1} - P_t$ so that $\Delta P = 0.1P_t^2 + 0.4P_t + 0.5 - P = 0.1P_t^2 - 0.6P_t + 0.5$. Then we solve the equation $\Delta P = 0$ to get the equilibrium points

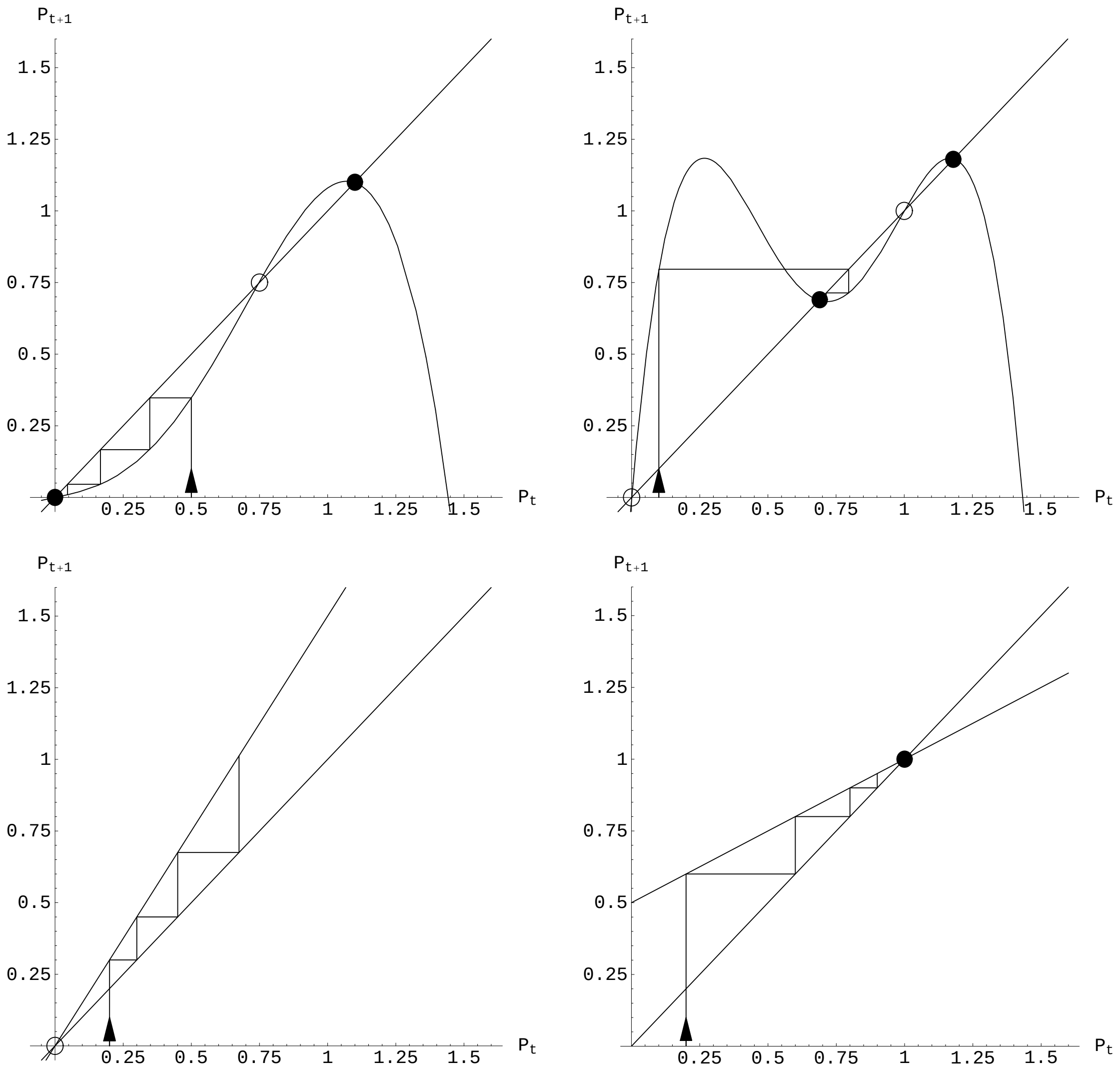
   $$\begin{aligned} \Delta P = 0 \;\Rightarrow&\quad 0.1P_t^2 - 0.6P_t + 0.5 = 0 \\ \Rightarrow&\quad P_t^2 - 6P_t + 5 = 0 \\ \Rightarrow&\quad (P-5)(P-1) = 0 \\ \Rightarrow&\quad P = 1 \quad \text{or} \quad P = 5. \end{aligned}$$

2. For each of the models in question 1 sketch a graph of ΔP vs P, and hence determine the stability of the equilibrium points.



From the graphs above we see that for (a) $P = 5$ is stable; for (b) $P = 0$ is unstable and $P = 8$ is stable; for (c) $P = 1$ is stable and $P = 5$ is unstable.

3. For each of the following graphs showing x_{t+1} vs x_t for some population label all the equilibrium points, indicate their stability, and draw a cobweb diagram showing the population for four time steps starting at the point indicated



The equilibria occur where the function crosses the line $y = x$. These are indicated with dots on the graphs, with a solid dot being stable (cobwebs would approach it) and a circle being unstable (cobwebs would go away from it). When drawing cobwebs be sure to go up to the function first to get your output and then across to the line $y = x$ to make your output the next input.

4. In the absence of predation, a population of fish grows according to the logistic difference equation

$$\Delta P = 0.2P(1-P)$$

where P is measured in units so that the carrying capacity is 1. Suppose fish are harvested at a rate proportional to the number of fish so that the new difference equation is

$$\Delta P = 0.2P(1-P) - hP$$

where h is a proportionality constant indicating the rate of harvesting.

(a) What is the biologically meaningful range of values for h?
h is the per capita rate at which the population is removed, so h can be no larger than 1 (you can't remove more than 100%) and h must be positive, otherwise you are not removing fish.

(b) Challenge: Find the equilibrium points for the population with harvesting, expressing your answer in in terms of h.
Set $\Delta P = 0$ and solve for P. This gives

$$0.2P(1-P)-hP = 0 \;\Rightarrow\; P(0.2-h-0.2P) = 0 \;\Rightarrow P = 0 \quad \text{or} \quad P = \frac{0.2-h}{0.2} = 1-\frac{h}{0.2} = 1-5h\;.$$

The second equilibrium value is reduced form the carrying capacity of 1 for positive values of h.

(c) Determine the harvesting level for which the only equilibrium is at $P = 0$. This will happen when the second fixed point is $P = 0$, which occurs when $5h = 1 \;\Rightarrow\; h = 0.2$. This makes sense since the intrinsic growth rate is 0.2.

(d) Challenge: Determine the stability of the equilibria (Hint: this will depend on the value of h is larger or smaller than the value calculated above.).
For stability we look at the second fixed point $P = 1 - \frac{h}{0.2}$. When this is bigger than the fixed point $P = 0$, then $P = 0$ is unstable and $P = 1 - \frac{h}{0.2}$ is stable. This happens when $h > 0.2$. For $h < 0.2$ the second fixed point is negative (and unphysical). The fixed point at $P = 0$ becomes stable – all solutions tend to $P = 0$.

5. For the following non-linear model

$$P_{t+1} = 5P_t - P_t^3$$

   (a) Write down an expression for the per capita growth rate $\Delta P/P$.
   First we find

$$\Delta P = P_{t+1} - P_t = 5P_t - P_t^3 - P_t = 4P_t - P_t^3\,.$$

   Therefore

$$\frac{\Delta P}{P} = \frac{4P - P^3}{P} = 4 - P^2\,.$$

   (b) What is intrinsic growth rate (ie the per capita growth rate at $P = 0$)?
   The intrinsic growth rate is 4.

   (c) Find the equilibrium population.
   This happens when $4 - P^2 = 0$, or when $P^2 = 4 \Rightarrow P = 2$.

6. Each of the following difference equations represents logistic growth in a different way. For each model find the intrinsic growth rate and the carrying capacity. (Hint: for the second one you may need to do some manipulation to get it into the right form) that the per capita growth rate for the logistic model should be linear and of the form $\mu(1 - P_t/K)$, so calculate the per capita growth rate for each of these models and express them in the correct form.

   (a) $P_{t+1} = P_t + 0.1P_t(1 - P_t/10)$
   The general form of the logistic model is $P_{t+1} = P_t + rP_t(1 - P_t/K)$. The given equation is in the right form so we read that the intrinsic growth rate is $r = 0.1$ and the carrying capacity is $K = 10$.

   (b) $P_{t+1} = P_t + 0.2P_t(4 - P_t)$ For this problem we need to put the equation in the right form. Notice that the quantity in the parenthesis is $(4 - P_t)$ is not in the form $1 - \frac{P_t}{K}$. However, we can divide and multiply by 4 to get

$$(4 - P_t) = \frac{4(4 - P_t)}{4} = 4(1 - \frac{P_t}{4})\,.$$

   Hence, the difference equation is

$$P_{t+1} = P_t + 0.2P_t(4 - P_t) \Rightarrow P_{t+1} = P_t + 0.2(4)P_t(1 - \frac{P_t}{4}) \Rightarrow P_{t+1} = P_t + 0.8P_t(1 - \frac{P_t}{4}).$$

   Thus $r = 0.8$ and $K = 4$.

   (c) $P_{t+1} = 1.3P_t - 0.05P_t^2$