Calculus II take-home final, Winter 2006
Due in class on Tuesday, 3/14/06

Instructions: *This is an open-book, open-note test. You may not use graphing or calculus features of your calculator. You also should not work with anyone else on the test. The questions are divided into three categories: straightforward, medium, and more involved. This is just a rough guide, and you may find some problems more or less difficult than I expect. Everyone should attempt to do all the problems, and show how far you got in each one.*

Straight-forward problems

1. Calculate the following integrals
   (a) $\int -\sin(z-8)\,dz$
   (b) $\int \frac{54x}{\sqrt{x^5}}\,dx$
   (c) $\int_1^2 \frac{1+y^2}{y}\,dy$
   (d) $\int_0^4 e^{2t}\,dt$
2. Calculate the area of the region between the curves $y = 2x^3$ and $y = 16$ and $x = 0$
3. Find the global maximum and minimum values of $f(x) = x^3 - 3x^2 + 2x$ on $[0, 4]$.
4. Calculate the following limits if they exist. (Do not use a calculator.)
   (a) $\lim_{x\to 0} \frac{-3}{x^2}$
   (b) $\lim_{y\to 0} \frac{sinx}{x^2}$
   (c) $\lim_{x\to 0} \frac{e^x}{x^{200}}$
5. Find the linear approximation of $y = e^x$ at $x = 1$

Medium-level problems

1. Evaluate the following integrals.
   (a) $\int_0^{\sqrt{\pi}} x^3 \sin(x^2)\,dx.$
   (b) $\int_{-2}^4 |\,x+1|\,dx.$
   (c) $\int_5^{10} \frac{x}{\sqrt{x-1}}\,dx.$
   (d) $\int \cos^2(2x-1)\,dx$
   (e) $\int \frac{4x}{e^x}\,dx$
   (f) $\int_0^{\pi/2} \cos^5 x\,dx.$

2. Sketch an accurate graph of the function $g(x) = xe^{-x^2}$. This should be accomplished by calculating the location of critical points, identifying intervals on which the function is increasing or decreasing or concave up or down, and identifying any points of inflection.

3. Write a parameterization for a circle of radius 3 centered at the point (-3,2) and traced out clockwise.

4. Calculate the following limits if they exist. Do not use a calculator.

   $\lim_{t \to \pi} \frac{sin^2 t}{t - \pi}$

   $\lim_{x \to 0} \frac{sin 2x - 2 sin x}{2 tan x - tan 2x}$

5. An object is moving along the y-axis, starting at $t = 0$. The velocity of the object at time t seconds is $v(t) = t^2 - 4t$ inches/second.

   (a) Where is the object (relative to its starting position) after 6 seconds?

   (b) What is the total distance travelled by the object during the first 6 seconds?

6. Use Simpson's rule with $n = 4$ subintervals to approximate the length of the curve $y = \sqrt{x}$, for $1 \leq x \leq 2$.

More involved problems

1. The graph of $y = \tan\left(\frac{\pi}{3}x^4\right)$ between $x = 0$ and $x = 1$ is rotated around the y-axis to form a container that holds water. Units of length are in meters. Water is being poured into the container.

   (a) Let $V(h)$ be the volume of water in the container when the the water level is h meters above the x-axis. Express $V(h)$ as a definite integral with respect to y. *Do not evaluate the integral.*

   (b) Compute $\frac{dV}{dh}$ as a function of h.

   (c) Suppose that water is poured into the container at a constant rate of $\frac{dV}{dt} = 2$ cubic meters per second. Find $\frac{dh}{dt}$, the rate at which the water level is increasing, when $h = 1$ meter.

2. A cruise ship moving at 15 miles/hour uses fuel at a cost of \$800/hour. Furthermore, the cost of fuel is proportional to the cube of the speed. The additional cost of running the cruise ship, including staffing, booze, and shuffle-board maintenance is \$2000/hour. You own the cruise ship, and you are also the captain. At what speed should you travel so that the cost of a trip from Olympia to Acapulco is minimized? (You do not need to know how far it is from Olympia to Acapulco.)

3. Calculate the volume of a jelly donut. You may use the suggestions on the solutions to Workshop #4, or devise another method of your own. Make sure all of your assumptions and dimensions are explicitly given. (You will probably run across a trig substitution in this problem: use the solutions to Workshop 4 as a model for this part of the calculation.)