1. All second order linear homogenous differential equations can be expressed in the *standard form*

$$y'' + b(x)y' + c(x)y = 0$$

Show that if we multiply this equation by an appropriate integrating factor $p(x)$ to get

$$p(x)y'' + p(x)b(x)y' + p(x)c(x)y = 0$$

we can transform the original differential equation into Sturm-Liouville form

$$\frac{d}{dx}\left(p(x)\frac{dy}{dx}\right) + q(x)y + \lambda\sigma(x)y = 0\,,$$

provided that $p(x)$ satisfies the differential equation $p'(x) = b(x)p(x)$ and that we identify $q(x) + \lambda\sigma(x)$ with $p(x)c(x)$. Show that this means the appropriate integrating factor is

$$p(x) = e^{\int b(x)dx}$$

Use this idea to express the following equations in Sturm-Liouville form And identify the Sturm-Liouville functions $p(x)$, $q(x)$ and the weight function $\sigma(x)$. Given the requirement that $p(x) > 0$ and $\sigma(x) > 0$, state any restriction on x. (Note you may have to first express them in *standard form* before using the method described above.)

(a) Hermite's equation $y'' - 2xy' + \lambda y = 0$

(b) Bessel's equation $x^2y'' + xy' - y + \lambda x^2 y = 0$

(c) Chebyshev's equation $(1 - x^2)y'' - xy' + \lambda y = 0$