1. If $u = x^2y - xy^2$ and $x = e^s \cos t$ and $y = e^s \sin t$ find $\frac{\partial u}{\partial s}$ and $\frac{\partial u}{\partial t}$ at the point $s = 0$ and $t = \frac{\pi}{4}$

2. Suppose a cylindrical tree trunk grows in radius at a rate of 0.5 cm per year, and grows in height at a rate of 8 cm per year. Find the rate at which its volume grows per year when its radius is 4 cm and its height is 200 cm.

3. Find the directional derivative of the function $f(x, y) = e^{xy} - x^2$ at the point $(1, 1)$ in the direction of the vector $\vec{v} = \vec{i} + 2\vec{j}$. Also find the direction of the steepest slope and the magnitude of the slope in that direction.

4. Suppose the temperature in Fahrenheit on a 2 ft by 2 ft square concrete wall is given by the expression $T = 10x^2y + 40$, where x and y are horizontal and vertical coordinates measured in feet starting from the bottom left corner of the square. A small snail moves along the concrete in the direction of the temperature gradient $\vec{\nabla}T$.

   (a) If the snail starts off half way along the bottom edge at (1,0) find its initial direction of motion.

   (b) The snail travels along a path described by some function $y = f(x)$. Show that the snail follows the temperature gradient the slope of this function is given by given by $\dfrac{\mathrm{d}y}{\mathrm{d}x} = T_y/T_x$. Hence show that the path of the snail satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{2y} \, .$$