1. Compute the following double integrals

(a)

$$I = \int\int_{[1,2]\times[0,1]} x\sqrt{1+y+x^2}dA$$

In this case the order you choose for integration won't make a big difference. Let's do the $x$ integration first

$$I = \int_0^1\int_1^2 x\sqrt{1+y+x^2}dxdy$$

Now we need to evaluate $\int x\sqrt{1+y+x^2}dx$. We'll do this by substitution. Let $u = 1+x^2+y$ then, since we keep $y$ constant. $du = 2xdx \Rightarrow dx = du/2x$. Thus

$$\int x\sqrt{1+y+x^2}dx = \int x\sqrt{u}du/2x = \tfrac{1}{2}\int\sqrt{u}du = \tfrac{1}{2}\frac{u^{3/2}}{\frac{3}{2}} = \frac{u^{3/2}}{3} = \frac{(1+y+x^2)^{3/2}}{3}.$$

So our double integral reduces to the single integral

$$I = \frac{1}{3}\int_0^1 \left[(1+y+x^2)^{3/2}\right]_1^2 dy = \frac{1}{3}\int_0^1 (5+y)^{3/2} - (2+y)^{3/2}dy$$

which then becomes

$$I = \frac{1}{3}\left[\frac{(5+y)^{5/2}-(2+y)^{5/2}}{\frac{5}{2}}\right]_0^1 = \frac{2}{15}\left((6)^{5/2}-(3)^{5/2}-(5)^{5/2}+(2)^{5/2}\right)$$

.

(b)

$$\int\int_{[0,1]\times[1,2]}\frac{x}{x+y}dA$$

The $y$ integral looks easiest so lets do that first

$$I = \int_0^1\int_1^2\frac{x}{x+y}dydx = \int_0^1 \left[x\ln|x+y|\right]_1^2 dx = \int_0^1 x\ln(2+x) - x\ln(1+x)dx$$

Now we are left with single integrals of the form $\int x\ln(x+a)$, which we can either look up in a table, do with technology, or do by parts. I get

$$\int x\ln(x+a) = \frac{ax}{2} - \frac{x^2}{4} - \tfrac{1}{2}a^2\ln(x+a) + \tfrac{1}{2}x^2\ln(x+a)$$

so that

$$\begin{aligned} I &= \left[\frac{2x}{2} - \frac{x^2}{4} - \tfrac{1}{2}2^2\ln(x+2) + \tfrac{1}{2}x^2\ln(x+2) - (\frac{x}{2} - \frac{x^2}{4} - \tfrac{1}{2}\ln(x+1) + \tfrac{1}{2}x^2\ln(x+1))\right]_0^1 \\ &= \tfrac{1}{2} - \frac{3}{2}\ln 3 + 2\ln 2 \end{aligned}$$

(c)

$$\int\int_{[0,1]\times[1,2]} ye^{xy} dA$$

It looks like it might be best to do the $x$ integral first

$$\begin{aligned} I &= \int_1^2 \int_0^1 ye^{xy} dy dx = \int_0^1 \left[ y\frac{e^{xy}}{y} \right]_0^1 dy = \int_1^2 \left[e^{xy}\right]_0^1 dy \\ &= \int_1^2 (e^y - 1) dy = [e^y - y]_1^2 = (e^2 - 2 - e + 1) = e^2 - e - 1 \end{aligned}$$