1. Let V be $C[0,1]$ with inner product $< f \,|\, g >= \int_0^1 f(t)g(t)\,dt$.

   (a) Find $||t||$.

   (b) Let W be the subspace of V spanned by $f(t) = t$. Given $g(t) = t^2$, find the projection of g onto W. That is find $g_{||} = \text{proj}_W g = \dfrac{< g|f >}{< f|f >} f(t)$.

   (c) Find $g_{\perp} = g - g_{||}$ and verify that $< g|g_{\perp} >= 0$.

   (d) According to the Best Approximation Theorem $g_{||}$ is the best approximation to g in W. Find the error in this approximation. That is, find $\text{d}(g, g_{||})$,

2. Let V be the subspace of $C[-1,1]$ spanned by $\{1,t\}$ with inner product $< f \,|\, g >= \int_{-1}^{1} f(t)g(t)\,dt$.
Given $f(t) = 1 - t$, find a function in V which is orthogonal to f.

3. Let W be the subspace of $C[0,1]$ spanned by $\{1,t\}$ with inner product $< f \,|\, g >= \int_{0}^{1} f(t)g(t)\,dt$.
Given $f(t) = 1 - t$, find a function in W which is orthogonal to f.