

# Non-Linear Dynamics Homework Solutions

## Week 8

March 8, 2009

- 8.4.1** Consider the system  $\dot{r} = r(1 - r^2)$  and  $\dot{\theta} = \mu - \sin \theta$  for  $\mu$  slightly greater than 1. Sketch waveforms of  $x(t)$  and  $y(t)$ .

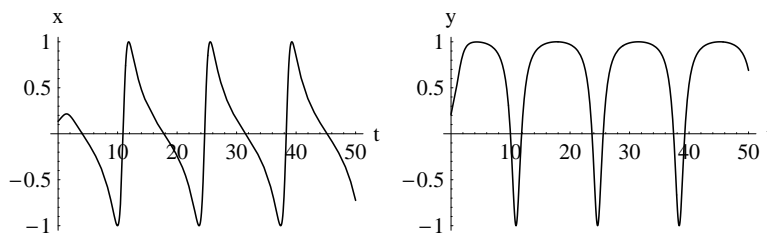


Figure 1: Plot of the waveforms of  $x(t)$  and  $y(t)$

- 8.4.2** Discuss the bifurcations of the system  $\dot{r} = r(\mu - \sin r)$ ,  $\dot{\theta} = 1$  as  $\mu$  varies.

This system has no fixed points, but  $\dot{r}$  can take on zero values when  $|\mu| \leq 0$ . When equality holds, an infinite number of fold bifurcations occur, each separate from the next by  $2\pi$ . When  $\mu$  passes below or above either of these critical values, the bistable limit cycles which arise when equality holds, split apart into two limit cycles, one stable and one unstable. As these move apart they collide with those which arose from neighboring fold bifurcations and annihilate each other as  $|\mu|$  becomes greater than 1.

- 8.4.3** Using numerical integration, find the value of  $\mu$  at which the system

$$\begin{aligned}\dot{x} &= \mu x + y - x^2 \\ \dot{y} &= -x + \mu y + 2x^2\end{aligned}$$

undergoes a homoclinic bifurcation. Sketch the phase portrait just above and below the bifurcation.

Numerical plots of the system using Mathematica show us that  $.06 < \mu_c < .07$ . Phase portraits are shown in Figure 2.

- 8.5.1** Show that  $[\ln(I - I_c)]^{-1}$  has infinite derivatives of all orders at  $I_c$ .

We use the hint on the first couple of derivatives to get some intuition as to what the derivatives are doing. We find that

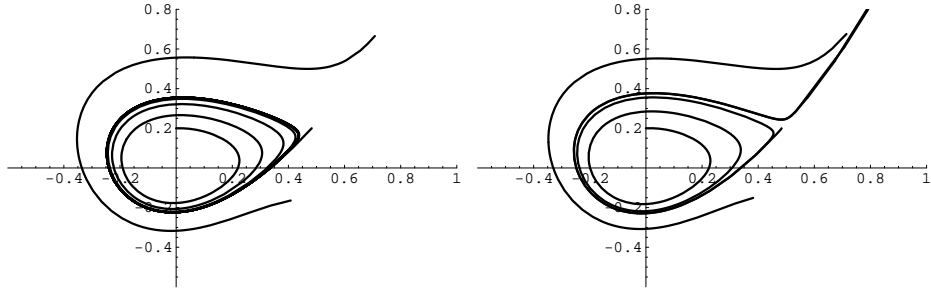


Figure 2: Numerical plots for  $\mu = .06$  and  $.07$  from left to right

$$\frac{d}{dt} \frac{1}{\ln[I - I_c]} = \frac{-1}{(I - I_c) \ln[I - I_c]^2}$$

$$\frac{d^2}{dt^2} \frac{1}{\ln[I - I_c]} = \frac{1}{(I - I_c)^2 \ln[I - I_c]} + \frac{2}{(I - I_c)^2 \ln[I - I_c]^3}$$

If we continue in this fashion we see that we will continue to get terms of  $(I - I_c)$  in the denominator of the summands involved in each derivative. Thus, each derivative is undefined at  $I = I_c$ .