Part I

Please answer all questions on the first part of this test, showing your work for full credit. You have one hour.

1. Suppose the position of a particle as a function of time is specified by polar coordinates

$$r = \tfrac{1}{2}\,at^2 \qquad\qquad \theta = \omega t\ .$$

   Find an expression for the acceleration as a vector function of time, giving your answer in polar form.

2. An object with mass m experiences a force F which depends on position x. The velocity is observed to vary with position as $v = \dfrac{k}{x}$. Find an expression for $F(x)$.

3. Given the unit tangent and radial vectors $\hat{\theta}$ and $\hat{r}$, show that

$$\frac{\mathrm{d}\hat{\theta}}{\mathrm{d}t} = -\dot{\theta}\hat{r}$$

4. A ball with mass m is thrown directly up with speed v_0. Find expressions for how long it takes to reach its maximum height, assuming

   (a) no air resistance.

   (b) linear air resistance with $f = -bv$.

5. The position as a function of time for an object falling under the influence of quadratic drag is given by the expression

$$y = \frac{v_{\text{ter}}^2}{g} \ln\left(\cosh\left(tg/v_{\text{ter}}\right)\right) .$$

   Use a Taylor expansion to show that for small t this expression reduces to the usual result for free fall with no air resistance. (Note: You may find the following Taylor series useful. $\cosh x = 1 + \frac{1}{2}x^2 + \frac{1}{4!}x^4 + \cdots$, and $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 + \cdots$,) Part II

This is the take home portion of the test. Please answer all questions, showing your work for full credit. You may consult your notes and the textbook but must not work with other people.

1. A boat is launched upstream into a river with initial velocity v_0 and experiences a resistive force in the opposite direction to its velocity given by $f = b\,e^{kv}$, where v is the speed of the boat, and b and k are constants.
   (a) Derive an expression for v as a function of time, in terms of m, b, k and v_0
   (b) Find how long the boat travels up stream before turning around.
   (c) Find how far it travels in that time (You may wish to use Maple to do the required integral.)

2. A water drop falls through a cloud and accumulates moisture.
   (a) For what radius does the quadratic drag become exactly equal to linear drag?
   (b) For what radius is the terminal velocity predicted by linear drag equal to the terminal velocity predicted by quadratic drag? (The density of water is 1000 kg per m^3.)

3. Tarzan (mass 70 kg) swings from a vine with length $\ell = 10$ m.
   (a) Find an expression for the angular acceleration as a function of angle θ measured counter clockwise from the position when the vine is vertical. Hence show that the angular acceleration is zero at the bottom of the swing.
   (b) If his angular speed at the bottom of his swing is 0.8 radians per second find his radial acceleration at the bottom of the swing and hence determine the force he exerts on the vine.