Please complete all questions

1. Consider the following curvilinear transformation of coordinates

$$x = e^u \cos v \qquad y = e^u \sin v \qquad z = w$$

(a) Show that (u, v, w) are an orthogonal coordinate system.

(b) Express the line element ds^2 in terms of u, v, w.

(c) Find the divergence of the vector field $\vec{F} = w\hat{e}_v - e^u\hat{e}_w$.

2. Let $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$ where

$$\vec{b}_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \qquad \text{and} \qquad \vec{b}_2 = \begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix}$$

be basis for a vector space.

(a) A vector $\vec{v}$ expressed in coordinates relative to the basis $\mathcal{B}$ is $[\vec{v}]_{\mathcal{B}} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

Express $\vec{v}$ in the standard basis.

(b) Another vector $\vec{x}$ expressed in the standard basis is $\vec{x} = \begin{pmatrix} 9 \\ 2 \\ 7 \end{pmatrix}$. Express this vector in coordinates relative to the basis $\mathcal{B}$.

3. Given the set of polynomials $\mathbb{P}_2$ on $C[-1, 1]$ with inner product $< f \,|\, g >= \int_{-1}^{1} f(t)g(t)\, dt$.

(a) Find $< 1|\, t^2 >$ and $||\, t^2||$.

(b) Show that the set $\mathcal{B} = \{t, t^2, 1 - \frac{5}{3}t^2\}$ is orthogonal on $C[-1, 1]$.

(b) Taking $\mathcal{B}$ as a basis for $\mathbb{P}_2$, find the coordinate vector for the polynomial $p(t) = 1 + t + t^2$ relative to the basis $\mathcal{B}$

4. Given the set of vectors

$$S = \left\{ \begin{pmatrix} -1 \\ 3 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 6 \\ -8 \\ -2 \\ -4 \end{pmatrix}, \begin{pmatrix} 6 \\ 3 \\ 6 \\ -3 \end{pmatrix} \right\}$$

Use Gram Schmidt to find an orthogonal basis for Span(S).