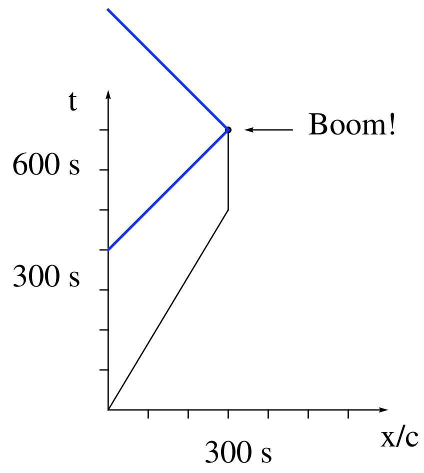
1.

The space time diagram on the right shows the world-line of a rocket which leaves earth, travels for a certain amount of time, comes to rest for short time and then explodes.



(a) What are the spacetime coordinates of the three events?

The rocket leaves earth at (0,0) comes to rest at (300,500) and explodes at (300,700).

(b) What was the speed of the rocket relative to the earth before it came to rest?
Speed is $\Delta x/\Delta t = 300/500 = 0.6c$

(c) How long was the rocket moving before it came to rest? (Answer both according to earth observers and according to rocket observers?)
According to the earth observer 500 seconds, according to the people in the rocket $\Delta t_p = \sqrt{500^2 - 300^2} = 400$ seconds.

(d) A light signal from earth reaches the rocket just as it explodes. Draw the worldline of this light signal. Also draw the world line of the light from the explosion. When do earth observers see the explosion?
They see the explosion 1000 seconds after the rocket leaves which is 300 seconds after they say it exploded.

2. (a) Electrons are accelerated from rest through a potential difference of 1.8×10^5 V. Calculate as measured in the laboratory reference frame the

(i) total energy of the electron in MeV.
Electron gains kinetic energy $KE = 1.8 \times 10^5$ eV$= 0.18 MeV$.
Total energy $E = KE + E_R = 0.18 + 0.511 = 0.69 MeV$

(ii) electron velocity.

$$E = \gamma mc^2 = \gamma E_R \;\Rightarrow\; \gamma = \frac{E}{E_R} = \frac{0.69}{0.511} = 1.35.$$

$$\text{So } \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}} = 0.67 \;\Rightarrow\; v = 0.67c$$

(b) If another beam of electrons where to be accelerated through the same potential difference but in the opposite direction to the first beam what would be the relative velocity of an electron in one beam with respect to the other beam?

Using relativistic addition of velocities $v' = \dfrac{v - u}{1 - vu/c^2} = \dfrac{0.67c - (-0.67c)}{1 - (0.67c)(-0.67c)/c^2} = 0.92c$

3. Use the microscope equation in the form

$$f(x) \approx f(0) + f'(0)x$$

when x is small to show that the function

$$f(x) = \frac{1}{\sqrt{1-x}}$$

can be expressed as

$$f(x) \approx 1 + \tfrac{1}{2}\, x$$

when x is small.

$f(x) = (1-x)^{-\frac{1}{2}}$ so $f'(x) = -\frac{1}{2}\,(1-x)^{-\frac{3}{2}}(-1)$ so $f'(0) = \frac{1}{2}$. Therefore $f(x) \approx 1 + \frac{1}{2}\,x$

Use this result to find and simplify an approximate expression for

$$E = mc^2\gamma = \frac{mc^2}{\sqrt{1-(v/c)^2}}$$

when $(v/c)^2$ is small. Hence show that KE $\approx \frac{1}{2}\,mv^2$ at nonrelativistic speeds.

Take $(v/c)^2$ as x. Then $\dfrac{mc^2}{\sqrt{1-(v/c)^2}} \approx mc^2(1 + \frac{1}{2}\,(v/c)^2) = mc^2 + \frac{1}{2}\,mv^2$. The first term is the rest energy and the second term is the kinetic energy.

4. UPS plans a new Super-Express mail system to compete with email. They wish to send 25 g letters at 0.999c around the world. How much would it cost to send one letter? (Electricity costs \$0.04 per 10^6 Joules). What would happen to the person receiving such a letter? (Hint: A nuclear bomb releases 5×10^{14} J.

The kinetic energy of such a letter would be $mc^2(\gamma - 1)$. For $v = 0.999c$ we have $\gamma = 22.4$ so the kinetic energy is $(0.025)(3 \times 10^8)^2(22.4 - 1) = 4.8 \times 10^{16}$ Joules. So it costs $(\$0.04)(4.8 \times 10^{16})/(10^6) = \$1.9 \times 10^{15}$. Kind of pricey. The energy of the letter is equivalent to the energy released by 100 nuclear bombs, so it might hurt a tad to catch.