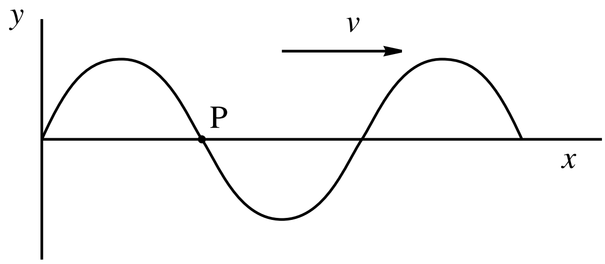
1. The diagram on the right shows a wave at a particular moment in time as it travels along a rope in the direction shown. Which one of the following statements is true about the point P on the rope?



   (a) It is moving upwards.

   (b) It is moving downwards.

   (c) It is moving to the right.

   (d) It is momentarily at rest.

   Answer (a) Although it is at equilibrium position it moving upwards at maximum speed

2. A standing wave is generated on a string which is fixed at both ends, and vibrates at its fundamental frequency. The tension of the string is now increased and a new standing wave vibrating at its fundamental frequency is generated. Which one of the following statements about the change in the properties of the wave is correct

   (a) The wave speed increases and the wavelength increases.

   (b) The wave speed increases and the frequency increases.

   (c) The wave speed decreases and the wavelength decreases.

   (d) The wave speed decreases and the frequency decreases.

   Answer (b). Increasing tension increases the speed. Since the fundamental frequency is proportional to the speed then the fundamental frequency will also increase. The wavelength will be twice the length of the string and this has not changed.

3. A wave of frequency 5.0 Hz travels along a string with a speed of 20 m/s. The phase difference between the oscillations of the string separated by 1.0 m along the wave is

   (a) $\pi/4$ (b) $\pi/2$ (c) π (d) 2π

   Answer (b). The wavelength is $20/5 = 4$ m. 1 m separation is one quarter of a wavelength which is 90^0 or $\pi/2$ out of phase.

4. Two strings, one thick and the other thin, are connected to form one long string. A wave travels along the string and passes the point where the two strings are connected. Which of the following does not change at that point:

   (a) frequency

   (b) propagation speed

   (c) amplitude

   (d) wavelength

   Answer (a). Frequency depends only on the source. The speed changes in a new medium and as a result all other variables will change.

5. In a standing wave
   (a) the nodes are positions of maximum amplitude.
   (b) all points of the wave vibrate with the same amplitude.
   (c) the distance between successive nodes is one wavelength.
   (d) all the points between a pair of nodes vibrate in phase.

   Answer (d).

6. Two sinusoidal waves travel in the same medium but one with twice the wavelength of the other. Which of the following statements is true? The wave with the longer wavelength has
   (a) higher speed.
   (b) lower speed.
   (c) higher frequency.
   (d) lower frequency.

   Answer (d). The medium remains the same so the speed is unchanged and hence the frequency must decrease

Part II

1. The wave function of a wave traveling on a string is given by

$$y = 0.1 \sin\left(\tfrac{20}{17}\pi x - 200\pi t\right)$$

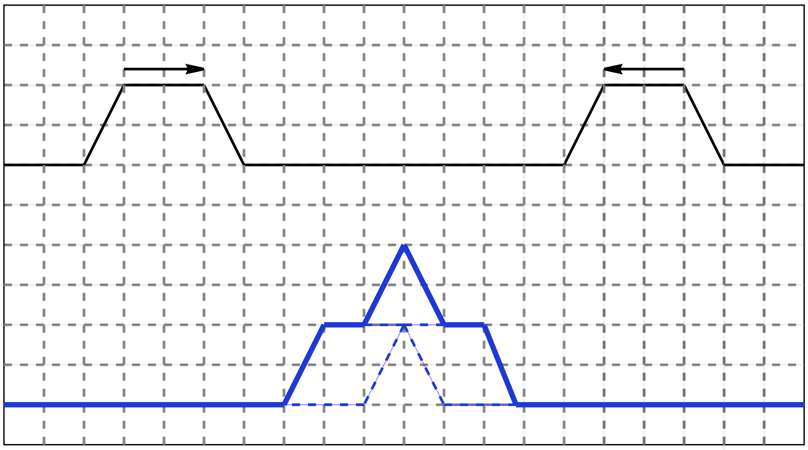
where y is the displacement in millimeters, t is time in seconds and x is the distance from the origin O in meters. Find

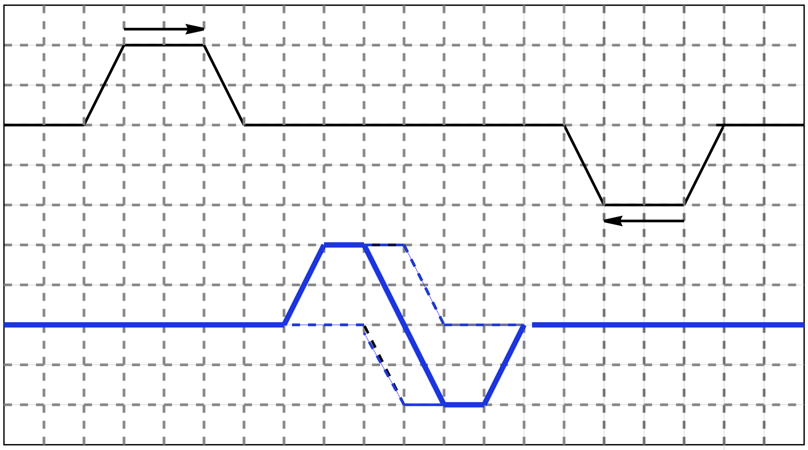
   (a) the frequency of the wave in Hertz,
   $f = \dfrac{\omega}{2\pi}$ and $\omega = 200\pi$ so $f = 200$ Hz
   (b) the wave length in meters,
   $\lambda = \dfrac{2\pi}{\kappa}$ and $\kappa = \tfrac{20}{17}\pi$ so $\lambda = 1.7$ m
   (c) the wave speed in meters per second,
   $v = \lambda f = 170$ m/s
   (d) the phase difference in radians between a point 0.25 m from O and a point 1.10 m from O,
   The distance form $x = 0.25$ m to $x = 1.10$ m is 0.85 m which is half a wave length so the phase difference is 180^o or π radians.
   (e) the wave function of a wave traveling in the same medium with double the amplitude and double the frequency but traveling in the opposite direction.

$$y = 0.2 \sin\left(\tfrac{40}{17}\pi x + 400\pi t\right)$$

   Note since the wave speed remains constant the wave length must be reduced by half which means κ is doubled.

2. The wave pulses below travel along a string at 1 cm/s. Draw pictures of the string at a moment 5 seconds after the time shown.





3. It is observed that a pulse requires 0.1 s to travel from one end of a stretched string to the other. The tension in the string is provided by passing the string over a pulley to a weight with a mass 40 times the mass of the string.

   (a) What is the length of the string?

   Let L be the length of the string. Then the speed is $v = L/0.1$ The speed is also given by the expression $v = \sqrt{F/\mu}$ where μ is the linear mass density m/L where m is the mass of the string. and F is the tension provided by the weight which is $40mg$. Equating the two expressions for speed gives

$$v = \frac{L}{0.1} = \sqrt{\frac{40mg}{m/L}} = \sqrt{40gL} \Rightarrow \frac{L^2}{0.01} = 40gL \Rightarrow L = 0.4g = 3.92\,\mathrm{m}$$

   (b) What is the fundamental frequency of this piece of string?

   The fundamental frequency is $f = v/2L$ and $v = L/0.1$ so $f = (L/0.1)/2L = 5$ Hz.

4. Two periodic sinusoidal waves $f(x) = A\sin(\kappa x - \omega t)$ and $g(x) = A\sin(\kappa x + \omega t)$ travel in opposite directions in the same medium.

   (a) Use the trigonometric identities $\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$ to find a simplified expression for the linear superposition $f(x) + g(x)$.
   First expand $f(x)$ and $g(x)$ using the identity

$$\begin{aligned} f(x) &= A\sin(\kappa x - \omega t) = A\sin\kappa x\cos\omega t - A\sin\omega t\cos\kappa x \\ g(x) &= A\sin(\kappa x + \omega t) = A\sin\kappa x\cos\omega t + A\sin\omega t\cos\kappa x \end{aligned}$$

   So that $y = f(x) + g(x) = 2A\sin\kappa x\cos\omega t$

   (b) Find an expression for the velocity of the string as it oscillates up and down.
   The velocity of the particles in the string is given by the rate of change in position y with respect to time $v_p = \dfrac{\mathrm{d}y}{\mathrm{d}t} = 2A\sin\kappa x(-\sin\omega t)\omega = -2A\omega\sin\kappa x\sin\omega t$

   (c) For which values of x is the string stationary?
   The string is stationary when $\dfrac{\mathrm{d}y}{\mathrm{d}t} = 0$. This is where $\sin\kappa x = 0$ which occurs when $\kappa x = n\pi$ where n is any integer. So $x = n\pi/\kappa = n\pi/(2\pi/\lambda) = n\lambda/2$. In other words, the nodes occur at multiples of half a wavelength.

5. Two periodic sinusoidal waves $f(x) = A\sin(\kappa x - \omega t)$ and $g(x) = A\sin(\kappa x - \omega t + \phi)$ meet in the same medium.

   (a) Use the half angle formula $\sin A + \sin B = 2\sin\left(\dfrac{A+B}{2}\right)\cos\left(\dfrac{A-B}{2}\right)$ to find a simplified expression for the linear superposition $f(x) + g(x)$.
   Using the half angle formula

$$\begin{aligned} y = f(x) + g(x) &= 2A\sin\left(\frac{(\kappa x - \omega t) + (\kappa x - \omega t + \phi)}{2}\right)\cos\left(\frac{(\kappa x - \omega t) - (\kappa x - \omega t + \phi)}{2}\right) \\ &= 2A\sin(\kappa x - \omega t + \phi/2)\cos(-\phi/2) \\ &= 2A\cos(\phi/2)\sin(\kappa x - \omega t + \phi/2) \end{aligned}$$

   (b) For which value of the phase difference ϕ is the amplitude zero.
   The amplitude is zero when $\cos(\phi/2) = 0$ which is when $\phi/2 = \pi/2$ so $\phi = \pi$. Which means when the waves are completely out of phase.

   (c) For which values of the phase difference ϕ is the amplitude maximum?
   The amplitude is maximum when $\cos\phi/2 = \pm 1$ which is when $\phi/2 = 0$, π, 2π etc. This is when ϕ is a multiple of 2π. So the amplitude is maximum when the two waves are in phase.