Part I

1. Given the function $f(x) = a^2 - x^2$ where a is a constant.

   (a) Find the area between the curve $y = f(x)$ and the x-axis from $x = 0$ to $x = a$ in terms of a.

   (b) The area between the curve $y = f(x)$ and the x-axis from $x = 0$ to $x = b$ is zero for one value of b. Find an expression for b in terms of a.

   (c) Write down an integral expression for the length of the curve $y = f(x)$ between $x = 0$ and $x = a$. Do not try to evaluate the integral.

   (d) The integral in (c) can be solved exactly by methods we will learn in Spring Quarter. It is difficult to use numerical methods to evaluate the integral since the constant a is not given. However it is possible to use numerical methods if we choose values for a. Write a program that uses Riemann sums to evaluate the integral accurate to 3 significant figures for each of the integer values of a between 0 and 5. Make sure you include a data table showing the value of a and the corresponding value of the integral that you calculated. Include your program with your solutions.

2. Although we have been using Euler's method to provide numerical solutions to the logistic equation

$$\frac{dP}{dt} = P\left(1 - \frac{P}{100}\right)$$

it turns out that there is analytical solution given by the formula

$$P(t) = \frac{100e^t}{1+e^t}$$

(a) Use algebra and the rules for differentiation to verify that this formula is in fact a solution to the logistic equation.

(b) Show that the following modified version of the formula,

$$P(t) = \frac{100Ae^t}{1+Ae^t}$$

with the extra parameter A is *also* a solution to the logistic equation for any value of A.

(c) Write a computer program to plot a graph of this solution with the parameter $A = 1/4$ on the interval $0 \le t < 10$. Include the program and plot with your solution.

(d) Write a computer program using Euler's method to graph an approximate solution on the interval $0 \le t < 10$ with an initial value of $P = 20$ and a stepsize $\Delta t = .005$. Include your program and plot with your solution.

(e) Show mathematically that using an initial value $P = 20$ corresponds to choosing $A = 1/4$.

Part II

The following questions may look familiar. They should. A brief scan of the in class test suggests that some of you did not have much time to complete these questions. Now is your chance to shine.

1. Differentiate the following functions. Show your work and simplify as far as possible

   (a) $f(x) = 3x^4 - 2\ln x - \dfrac{1}{x^2}$

   (b) $f(x) = x^2 e^{3x}$

   (c) $f(x) = \dfrac{x^2 - 1}{1 + x}$

   (d) $f(x) = \sqrt{1 - \sin^2 x}$

2. For the following rate equation find y as a function of x if $y' = \sin x - \cos x$ and $y = 2$ when $x = 0$.

3. Find the area under the curve $y = 12x - 3x^2$ between $x = 0$ and $x = 4$.