- This test forms part of our assessment of your personal learning in this program. You may not collaborate with other people, but you may consult your text book and notes.
- Attempt all questions on this test. Do not leave answers blank. Marks will be given for partial answers so show all your working.
- Your completed test is due at 9:00 am on Monday Feb 2nd.

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1. The Vanishing Elephants. According to one environmental group, the population of wild elephants declines by 5% per year. If E denotes the population t years from now, then we can express the decline by a rate equation of the form

$$E' = -kE$$

   (a) What value would you give k? (Think carefully)

   (b) Suppose the initial number of elephants is E_o. Write down an expression for the number of elephants after one year assuming that the rate is constant for an entire year. Now repeat the procedure for a second year with the rate for the second year. Generalize your result so that you have an expression for the number of elephants after n years.

   (c) If the initial population is 200,000 how many does your model predict will remain after 10 years. How many years will it take for the population to be cut in half to 100,000?

   (d) Consider this argument: "Since 1/20th of the population disappears each year, in 20 years the population will vanish completely." Does your rate equation predict that the elephant population will vanish in 20 years? What, if anything, is wrong with the argument quoted in the first sentence?

(e) In your model above you assumed that the rate was constant over an entire year. Now refine your model to correct this problem by using the rate equation with smaller step sizes. Starting with 200,000 elephants find out how many elephants there will be after 10 years. You should use a computer for this problem and decrease your step size until you have are accurate to the nearest 100 elephants.

(f) With your new model how long does it take for the population to reduced by half?

2. The arms race model you looked at in the in class test was developed by L.F. Richardson in 1920's as a way to explain the arms build up that lead to the first world war. He considered the model

$$\begin{aligned} x' &= -4x + 2y \\ y' &= 5x - 4y + 12 \end{aligned}$$

where x and y are the annual military budgets of the two countries (in billions of dollars)

(a) The following questions explore explore the meaning of the coefficients in these rate equations. To answer the questions it might be helpful to consider situations where either x or y or both are zero.

(i) What are the units of each of the coefficients?

(ii) Explain why it makes sense for the coefficient of x in the first equation and the coefficient of y in the second equation to be negative?

(iii) Explain why it makes sense for the coefficient of y in the first equation and the coefficient of x in the second equation should be positive?

(iv) What does the additional constant 12 in the second equation tell you about the military spending habits of country Y.

(b) In the in class test you used the model to predict the spending in one year based on a spending of $x = 5$ and $y = 6$ this year. You should have arrived at the answer $x = -3$ and $y = 19$. You may have worried about what $x = -3$ means. Infact this is an artifact of the approximation that the rate at which spending changes is constant during the entire year.

(i) Improve your estimate of the budgets in 1 year by taking two half year steps. Do this by hand. Is the estimate any better?

(ii) Improve your estimate again by taking 4 quarter year steps. Again do this by hand. Are your answers approaching a limit?

(iii) To find a good estimate for the budgets in one year you should make much smaller steps. Write a simple program to find the expected budget in one year which is accurate to 2 decimal places.

(c) Now you are ready to explore what the model predicts in 10 years. Using the same step size as in (b)(iii) adjust your model so that you can draw a graph of y vs x for t between 0 and 10 years. What are the final values of x and y after 10 years? Are they much different than they were after 1 year?

(d) Repeat the above procedure for different initial values of x and y. Use

(i) $x = 0$ $y = 0$

(ii) $x = 8$ $y = 10$

(iii) $x = 2$ $y = 12$

What is similar about the behaviour of the model in each of these cases? Explain this behaviour based on the original rate equations.