11.1 Using Fig. 11.2, verify that the change in the Sun’s effective temperature over the past 4.5 billion years is consistent with the variations in its radius and luminosity.

Recall $L = \frac{4\pi\sigma}{3} R^2 T^4 \rightarrow T = \frac{L}{\frac{4\pi\sigma}{3} R^2}$

Let $T_i =$ temp 4.5 billion years ago, read from chart.

Calculate $T_i = \frac{R_i}{R_0} \cdot \frac{L_i}{L_0}$

If $T_0 = 5770 K$ now, then $T_i =$

Compare to $T_i$ on chart =
11.2 (a) At what rate is the Sun's mass decreasing due to nuclear reactions? Express your answer in solar masses per year.
(b) Compare your answer to part (a) with the mass loss rate due to the solar wind.
(c) Assuming that the solar wind mass loss rate remains constant, would either mass loss process significantly affect the total mass of the Sun over its entire main-sequence lifetime?

\[ \text{Power} = \frac{\text{Energy}}{\text{time}} \]

\[ L = \frac{\Delta m}{c^2} \frac{\Delta t}{\Delta t} \]

\[ \text{Rate of mass change} = \frac{\Delta m}{\Delta t} \]

\[ \text{Ex. 1 found the solar wind causes a mass loss rate of} \ 4 \times 10^{-9} \frac{\Delta m}{\Delta t} \]

\[ \text{Compare:} \]

\[ \text{Sun's main sequence lifetime} \ 2 \times 10^{10} \text{ yrs} = \]

Total mass lost \( \Delta m = \Delta m \times \frac{\text{yr}}{\Delta t} \)
11.5 (a) Using Eq. (9.58) and neglecting turbulence, estimate the full width at half-maximum of the hydrogen Hilde absorption line due to random thermal motions in the Sun's photosphere. Assume that the temperature is the Sun's effective temperature.

\[
(\Delta \lambda)_{1/2} = \frac{2\lambda}{c} \sqrt{\left(\frac{2kT}{m} + v_{\text{turb}}^2\right) \ln 2},
\]

(9.58)

where mass per atom \( m = \mu_n = \mu_p = 1.67 \times 10^{-24}\) g

\( \lambda = \frac{\mu_n}{\mu_p} = 6562.80\) Å

\( T = 5770\) K \hspace{1cm} \( k = 1.38 \times 10^{-16}\) erg K

\[
\frac{2kT}{\mu} = \frac{2 \lambda}{c}
\]

\[
(\Delta \lambda)_{1/2}^2
\]

(b) Using H\(_\alpha\) redshift data for solar granulation, estimate the full width at half-maximum when convective turbulent motions are included with thermal motions.

If \( v_{\text{turb}} \approx 0.4\) km s\(^{-1}\)

Then \((\Delta \lambda)_{1/2}^2 = \)
(c) What is the ratio of $v_{\text{turb}}^2$ to $2kT/m$?

(d) Determine the relative change in the full width at half-maximum due to Doppler broadening when turbulence is included. Does turbulence make a significant contribution to $(\Delta \lambda)_{1/2}$ in the solar photosphere?

\[
\frac{v_{\text{turb}}^2}{2kT/m}
\]

Let \( \Delta \lambda_{\text{turb}} = (\Delta \lambda)_{1/2} \) with \( V_{\text{turb}} = 0.4 \text{ km/s} \), and \( \Delta \lambda_0 = (\Delta \lambda)_{1/2} \) with \( V_{\text{turb}} = 0 \).

Then \( \Delta \lambda_{\text{turb}} \cdot \Delta \lambda_0 = \) \( \Delta \lambda_0 \)
11.6 Estimate the thermally Doppler-broadened line widths for the hydrogen Lyman $\alpha$, C III, O VI, and Mg X lines given on page 401; use the temperatures provided. Take the masses of H, C, O, and Mg to be 1 u, 12 u, 16 u, and 24 u, respectively.

\[ \text{Eq. } (2.57) \ (\Delta \lambda)^2 = \frac{296}{296} \]

<table>
<thead>
<tr>
<th></th>
<th>Lyman $\alpha$</th>
<th>C III</th>
<th>O VI</th>
<th>Mg X</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_0$</td>
<td>H</td>
<td>C</td>
<td>O</td>
<td>Mg</td>
</tr>
<tr>
<td>$\text{mass}$</td>
<td>1</td>
<td>12</td>
<td>16</td>
<td>24</td>
</tr>
<tr>
<td>$\lambda (\AA)$</td>
<td>1216</td>
<td>977</td>
<td>1032</td>
<td>625</td>
</tr>
<tr>
<td>$T (K)$</td>
<td>20,000</td>
<td>90,000</td>
<td>300,000</td>
<td>$1.4 \times 10^6$</td>
</tr>
</tbody>
</table>
11.8 Suppose that you are attempting to make observations through an optically thick gas that has a constant density and temperature. Assume that the density and temperature of the gas are $2.5 \times 10^{-7} \text{ g cm}^{-3}$ and 5770 K, respectively, typical of the values found at the base of the Sun's photosphere. If the opacity of the gas at one wavelength ($\lambda_1$) is $\kappa_{\lambda_1} = 0.26 \text{ cm}^2 \text{ g}^{-1}$ and the opacity at another wavelength ($\lambda_2$) is $\kappa_{\lambda_2} = 0.30 \text{ cm}^2 \text{ g}^{-1}$, calculate the distance into the gas where the optical depth equals $2/3$ for each wavelength. At which wavelength can you see farther into the gas? How much farther? This effect allows astronomers to probe the Sun's atmosphere at different depths (see Fig. 11.17).

\[
(9.13) \quad d \tau_2 = \frac{260}{d}
\]

\[
d = \int ds = -\int_{\tau_{\lambda_1}}^{\tau_{\lambda_2}} \frac{1}{\kappa_{\lambda}} d\tau_2 = \]

Let $\tau_{\lambda_2} = \frac{2}{3}$. Then the depth observed at $\lambda$ is $d$:

For $\lambda_1$, $d_1 =$

For $\lambda_2$, $d_2 =$
11.9 (a) Using the data given in Example 11.2, estimate the pressure scale height at the base of the photosphere.

(b) Assuming that the mixing length to pressure scale height ratio is 2.2, use the measured Doppler velocity of solar granulation to estimate the amount of time required for a convective bubble to travel one mixing length. Compare this value to the characteristic lifetime of a granule.

\[ P = 5 \times 10^4 \, \text{dynes/cm}^2 \]
\[ \rho = 2.5 \times 10^{-3} \, \text{g/cm}^3 \]

Pressure scale height (10.63)

What is \( g \) at the surface of the sun (base of photosphere)?

\[ F = ma \]
\[ \frac{GmM}{r^2} = mg \]
\[ \frac{GM\bigodot}{R^2} = g = \]

1. So \( H_p = \)

2. Let \( \ell = \) mixing length

\[ \text{velocity} = \frac{\text{length}}{\text{time}} : V = \frac{\ell}{t} \to t = \]

\[ \text{If} \, V = 0.4 \times 10^3 \, \text{m/s}, \text{then convection time} \, t = \]
11.10 Show that Eq. (11.7) follows directly from Eq. (11.6). Assume \( T = \text{const} \) locally.

\[
\frac{d}{dr} \left( 2\pi kT \right) = -\frac{GM\Omega_n m_p}{r^2}
\]

\[
a \frac{dn}{dr} = b \frac{n(r)}{r^2} : a = \_, \quad b = \_ \]

\[
a \int \left[ \frac{dn}{n} \right] = b \int \frac{dr}{r^2} \quad \text{Solve both sides.}
\]

11.11 Calculate the magnetic pressure in the center of the umbra of a large sunspot. Assume that the magnetic field strength is 2000 G. Compare your answer with a typical value for the gas pressure at the base of the photosphere.

\[
\text{(11.3)} \quad \text{magnetic pressure} \quad P = \frac{B^2}{8\pi}
\]

\[
\text{Ex 11.2} \quad P_{\text{gas}} = \_ \]
11.12 Assume that a large solar flare erupts in a region where the magnetic field strength is 300 G and it releases $10^{32}$ ergs in one hour.

(a) What was the magnetic energy density in that region before the eruption began?

(b) What minimum volume would be required to supply the magnetic energy necessary to fuel the flare?

(c) Assuming for simplicity that the volume involved in supplying the energy for the flare eruption was a cube, compare the length of one side of the cube with the typical size of a large flare.

(d) How long would it take an Alfvén wave to travel the length of the flare?

(e) What can you conclude about the assumption that magnetic energy is the source of solar flares, given the physical dimensions and time scales involved?

\[ P = \frac{B^2}{8\pi} \]

\[ u_m = \frac{\text{Energy}}{\text{density}} \]

\[ \text{Volume} \sim x^3 \rightarrow x = \]

\[ \text{Alfvén speed} \quad v = \frac{B}{\sqrt{4\pi\rho}} \]

\[ v = \frac{x}{t} \rightarrow t = \]
11.13 (a) Calculate the frequency shift produced by the normal Zeeman effect in the center of a sunspot that has a magnetic field strength of 3000 G. = $B$

(b) By what fraction would the wavelength of one component of the $\lambda = 6302.5$ Å Fe I spectral line change due to a magnetic field of 3000 G?

\[
\Delta \nu = \frac{eB}{\hbar c}
\]

\[\text{shift (cgs)} = \frac{eB}{\hbar c}\]

$\Delta \nu = \frac{eB}{\hbar c}$

\[
\lambda = \frac{\lambda_0}{1 + \Delta \lambda / \lambda_0}
\]

\[
\Delta \lambda = \Delta \nu \lambda_0 = \frac{eB}{\hbar c} \lambda_0
\]

11.14 Argue from Eq. (11.12) and the work integral that magnetic pressure is given by Eq. (11.13).

From thermodynamics, work $W = \int P \, dV$

$P = \frac{\partial u}{\partial V}$

Therefore \[\frac{eB}{\hbar c}\] = \[\frac{B^2}{8\pi}\] (11.13)
\[ p = a T^x \]

\[ \ln p = \ln a T^x = \ln a + \ln T^x \]

\[ \ln p = \ln a + x \ln T \]

\[ \frac{d}{dt} \ln p = x \]

\[ p \propto T^2 \quad \text{convection} \]

\[ p \propto T^3 \quad \text{radiation} \]

\[ e^{x+y} = e^x e^y \]

\[ \ln ab = \ln a + \ln b \]

\[ e^{ab} = e^{\ln a + \ln b} \]

\[ ab = e^{\ln a} e^{\ln b} = ab \quad \checkmark \]

\[ \ln ab = \ln a + \ln b \]