13. (II) At what distance from the Earth will a spacecraft on the way to the Moon experience zero net force due to these two bodies because the Earth and Moon pull with equal and opposite forces?

\[ a + b = d = 3.84 \times 10^5 \text{ km} \]

**Force between Earth & Satellite:** \( F_a = G \frac{mM_e}{a^2} \)

**Force between Moon & Satellite:** \( F_b = G \frac{mmM_o}{b^2} \)

**Equal strengths:** \( F_a = F_b \)

Simplify \( G\frac{mM_e}{a^2} = G\frac{mmM_o}{b^2} \)

Sub in \( b = d - a \): \( \frac{M_e}{a^2} = \frac{b^2}{(d-a)^2} \)

Solve for \( a \):
28. (a) Show that if a satellite orbits very near the surface of a planet with period \( T \), the density (= mass per unit volume) of the planet is \( \rho = \frac{m}{V} = \frac{3\pi}{G T^2} \). (b) Estimate the density of the Earth, given that a satellite near the surface orbits with a period of about 90 minutes.

\[
F = \frac{G m M}{R^2} = \frac{m v^2}{R}
\]

\[
\frac{G M}{R} = v^2 = \left(\frac{2\pi R}{T}\right)^2 = \frac{4\pi^2 R}{T^2}
\]

\[
\frac{G M}{R^3} = \frac{4\pi^2 R}{T^2}
\]

and density \( \rho = \frac{m}{V} = \frac{4\pi R^3}{3\pi R^3} \)

\[
\frac{4}{3} \pi \rho = \frac{M}{R^3} = \frac{4\pi R^2}{G T^2}
\]

47. How far above the Earth's surface will the acceleration of gravity be half what it is at the surface?

\[
R = 6.4 \times 10^6 \text{ m}
\]

\[
F(d) = \frac{G m M}{d^2} = \frac{1}{2} \frac{G m m}{R^2}
\]

\[
d = h + R
\]
57. NASA launched the Near Earth Asteroid Rendezvous (NEAR), which, after traveling 1.3 billion miles, is meant to orbit the asteroid Eros at a height of about 15 km. Eros is potato-shaped: 40 km × 6 km × 6 km. Assume Eros has a density (mass/volume) of about $2.3 \times 10^3$ kg/m$^3$. (a) What will be the period of NEAR as it orbits Eros? (b) Suppose Eros to be a sphere with the same mass and density. What would its radius be? (c) What would g be at the surface of this spherical Eros?

Approximate Eros as a cylinder:

\[
\begin{align*}
\text{Volume} &= \text{area} \times \text{height} = \pi r^2 h \\
V &= \pi r^2 h \\
\text{Mass} &= \frac{\text{mass}}{\text{volume}} \times \text{volume} \\
&= \rho \times V
\end{align*}
\]
(1) If Eros was a sphere with the same mass and density, it would have: 

\[ V = \frac{4}{3} \pi R^3 \] 

\[ \text{mass} \quad \text{volume} \quad \rho \]

\[ R^3 = \frac{3 \text{mass}}{4 \pi \rho} = \frac{3 \left(2.65 \times 10^{15} \text{ kg}\right)}{4 \pi \cdot 2.13 \times 10^3 \text{ kg/m}^3} \]

\[ R = 6.5 \times 10^3 \text{ m} = 6.5 \text{ km} \]

(2) To find \( g \) at the surface of this sphere, recall that 

\[ F - mg = G \frac{Mm}{r^2} \]

\[ g = \frac{GM}{r^2} \]

\[ g = \frac{G M}{R^2} = \frac{6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \left(2.65 \times 10^{15} \text{ kg}\right)}{(6.5 \times 10^3 \text{ m})^2} = 4.2 \times 10^{-3} \text{ m/s}^2 \]
60. The Sun rotates about the center of the Milky Way Galaxy (Fig. 6–25) at a distance of about 30,000 light years from the center (1 ly = 9.5 \times 10^{15} \text{ m}). If it takes about 200 million years to make one rotation, estimate the mass of our Galaxy. Assume that the mass distribution of our galaxy is concentrated mostly in a central uniform sphere. If all the stars had about the mass of our Sun \((2 \times 10^{30} \text{ kg})\), how many stars would there be in our Galaxy?

\[
R = \frac{3 \times 10^4 \text{ ly}}{2 \text{ y}} = \frac{9.5 \times 10^{15} \text{ m}}{2 \text{ y}}
\]

\[
T = \frac{2 \times 10^8 \text{ yrs}}{2 \times 10^8 \text{ yrs}} = 1 \text{ yr}
\]

By Kepler’s 3rd law,

\[
M = \frac{4\pi^2 R^3}{GT^2}
\]

\[
galaxy's \ mass = \text{Number of stars} \times \text{mass per star}
\]

\[
M = N \times M_0
\]

\[
N = \frac{M}{M_0}
\]