Electromagnetism week 9

- Waves and wave equations
- Electromagnetism & Maxwell’s eqns
- Derive EM wave equation and speed of light
- Derive Max eqns in differential form
- Magnetic monopole → more symmetry

- Next quarter

Wave equation

1. Differentiate $\delta D/\delta t \rightarrow \delta^2 D/\delta t^2$
2. Differentiate $\delta D/\delta x \rightarrow \delta^2 D/\delta x^2$

3. Find the speed from

$$\frac{\delta^2 D}{\delta t^2} = \frac{\omega^2}{k^2} = \left(\frac{2\pi/T}{2\pi/\lambda}\right)^2 = \left(\frac{\lambda}{T}\right)^2 = (\lambda f)^2 = v^2$$

Waves

\[ D(x, t) = D_M \sin(kx - \omega t) \]

Causes and effects of E

Gauss: E fields diverge from charges
Lorentz force: E fields can move charges

\[ \oint E \cdot dA = \frac{q}{\varepsilon_0} \]
\[ F = q E \]
Causes and effects of B

Ampere: B fields curl around currents

Lorentz force: B fields can bend moving charges

\[ \oint B \cdot dl = \mu_0 I \]

F = q v x B = IL x B

Changing fields create new fields!

Faraday: Changing magnetic flux induces circulating electric field

\[ \frac{d\Phi_B}{dt} = -\int E \cdot dl \]

Guess what a changing E field induces?

Changing E field creates B field!

Current piles charge onto capacitor

Magnetic field doesn’t stop

Changing electric flux

\[ \Phi_E = \int E \cdot dA \]

→ “displacement current”

→ magnetic circulation

Partial Maxwell’s equations

Charge → E field

\[ \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0} \]

Current → B field

\[ \oint \vec{B} \cdot d\vec{l} = \mu_0 I \]

Faraday

Changing B → E

Ampere

Changing E → B

\[ \frac{d\Phi_B}{dt} = -\int E \cdot dl \]

\[ \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \oint B \cdot dl \]
Maxwell eqns → electromagnetic waves

Consider waves traveling in the x direction with frequency $f = \frac{\omega}{2\pi}$ and wavelength $\lambda = \frac{2\pi}{k}$

$E(x,t) = E_0 \sin (kx - \omega t)$ and $B(x,t) = B_0 \sin (kx - \omega t)$

Do these solve Faraday and Ampere’s laws?

Faraday + Ampere

\[ \frac{d\Phi_B}{dt} = -\int E \cdot dl \]
\[ \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} = \oint B \cdot dl \]

\[ \frac{dB}{dt} = -\frac{dE}{dx} \]
\[ \mu_0 \varepsilon_0 \frac{dE}{dt} = -\frac{dB}{dx} \]

Speed of Maxwellian waves?

Faraday: $\omega B_0 = k E_0$ \hspace{1cm} Ampere: $\mu_0 \varepsilon_0 \omega E_0 = kB_0$

Eliminate $B_0/E_0$ and solve for $v = \frac{\omega}{k}$

$\mu_0 = 4\pi \times 10^{-7}$ Tm/A \hspace{1cm} \varepsilon_0 = 8.85 \times 10^{-12}$ C$^2$/N/m$^2$
Maxwell equations → Light

\[ E(x,t) = E_0 \sin(kx-\omega t) \quad \text{and} \quad B(x,t) = B_0 \sin(kx-\omega t) \]

solve Faraday’s and Ampère’s laws.

Electromagnetic waves in vacuum have speed \( c \) and energy/volume = \( \frac{1}{2} \varepsilon_0 E^2 = \frac{B^2}{2\mu_0} \).

Full Maxwell equations in integral form

\[
\oint E \cdot d\vec{A} = \frac{q}{\varepsilon_0} \quad \oint E \cdot dl = -\frac{d\Phi_B}{dt} \\
\oint B \cdot d\vec{A} = 0 \quad \oint B \cdot dl = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt} + \mu_0 I
\]

Integral to differential form

**Gauss’ Law:**  \( \oint E \cdot d\vec{A} = \frac{q}{\varepsilon_0} \)  apply

Divergence Thm:  \( \oint \vec{v} \cdot d\vec{A} = \int (\nabla \cdot \vec{v}) \, d\tau \)  and the

Definition of charge density:  \( q = \int \rho \, d\tau = \int \frac{dq}{d\tau} \, d\tau \)  to find the

Differential form:

**Ampere’s Law:**  \( \oint B \cdot d\vec{l} = \mu_0 I \)  apply

Curl Thm:  \( \oint \vec{v} \cdot d\vec{l} = \int (\nabla \times \vec{v}) \cdot d\vec{A} \)  and the

Definition of current density:  \( I = \int J \, dA = \int \frac{dl}{dA} \, dA \)  to find the

Differential form:
Integral to differential form

**Faraday’s Law:** \[ \oint E \cdot dA = -\frac{d\Phi_B}{dt} = -\frac{d}{dt} \int B \cdot d\vec{A} \] apply

**Curl Thm:** \[ \oint \vec{v} \cdot d\vec{l} = \int (\nabla \times \vec{v}) \cdot d\vec{A} \] to find the

Differential form:

Finish integral to differential form...

**Faraday’s Law:** \[ \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0} \]

**Curl Thm:** \[ \oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt} \]

**Maxwell eqns in differential form**

\[ \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \]

\[ \nabla \times \vec{E} = -\frac{d\vec{B}}{dt} \]

\[ \nabla \cdot \vec{B} = 0 \]

\[ \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{d\vec{E}}{dt} + \mu_0 \vec{J} \]

Notice the asymmetries – how can we make these symmetric by adding a magnetic monopole?

Next quarter:

ElectroDYNAMICS, quantitatively, including

Ohm’s law, Faraday’s law and induction, Maxwell equations

Conservation laws, Energy and momentum

Electromagnetic waves

Potentials and fields

Electrodynamics and relativity, field tensors

Magnetism is a relativistic consequence of the Lorentz invariance of charge!