3.15. As noted in the chapter, the cosmic microwave background radiation fits the Planck equations for a blackbody at 2.7 K. (a) What is the wavelength at the maximum intensity of the spectrum of the background radiation? (b) What is the frequency of the radiation at the maximum? (c) What is the total power incident on Earth from the background radiation?

\[ \lambda_p T = 3 \times 10^{-3} \text{ m K} \]

(b) \[ E = \frac{hc}{\lambda} = hf \]

(c) Power = \[ 4\pi T^4 \] is incident on an effective area of a near-DISK of radius = \[ R_{\text{earth}} \] due to projection effects (more at equator, less at poles).
3.23. Use Planck's Law $u(\lambda) = \frac{8\pi\hbar c k^{-5}}{\lambda^5 (e^{\frac{\hbar c}{\lambda kT}} - 1)}$ (3.33) to derive the

constant in Wien's Law

peak $T = 3 \times 10^{-3}$ m.K (3.20)

Let $u = A\lambda^{-5} e^{\frac{\hbar c}{\lambda kT}} - 1$. We want to find where

$u$ peaks, or where $du = 0$.

Solve for $\lambda$.

$du = 0$
The orbiting space shuttle moves around Earth well above 99 percent of the atmosphere, yet it still accumulates an electric charge on its skin due to the loss of electrons caused by the photoelectric effect with sunlight. Suppose the skin of the shuttle is coated with Hf, which has a relatively large work function $\phi = 4.67$ eV at the temperatures encountered in orbit. (a) What is the maximum wavelength in the solar spectrum that can result in the emission of photoelectrons from the shuttle's skin? (b) What is the maximum fraction of the total power falling on the shuttle that could potentially produce photoelectrons?

**Photoelectric Effect**

Energy in = Energy out

$$\text{Light} = hf = \frac{1}{2} mv^2 + \phi$$

**If you turn on a repelling potential, it limits the maximum kinetic energy of emitted particles:**

$$eV = \left(\frac{1}{2} mv^2\right)_{\text{max}} = hf - \phi$$

(a) With no stopping potential, consider barely ejected electrons with $K E \rightarrow 0$. Then $hf = h\nu \rightarrow \phi$

$$\lambda = \frac{\lambda_{\text{max}}}{\phi}$$

(b) **Sun's Power Curve**

If $\lambda_{\text{max}}$ is the largest wavelength that can photo-eject electrons, then $f = \frac{\text{Power in sufficiently energetic photons}}{\text{Total incident power}} = \frac{\text{Shaded area}}{\text{Total area under curve}}$

$$\text{Total area} = R = \sigma T^4 = \int_{\lambda_{\text{max}}}^{\infty} u(\lambda) \lambda^2 \, d\lambda$$

Shaded area $\approx u(\lambda) \lambda^2 \, d\lambda$, where $d\lambda \sim \lambda_{\text{max}}$ and $u(\lambda)$
3.56. Derive (3.32) from (3.30) and (3.31) \( \rho138 \)

\[
\sum_{n=0}^{\infty} f_n = A \sum_{n=0}^{\infty} e^{-n\lambda T} = 1 \quad (3.30)
\]

The average energy of an oscillator is then given by the discrete-sum equivalent Equation 3-27,

\[
\bar{E} = \frac{\sum E_n f_n}{\sum f_n} = \frac{\sum E_n A e^{-E_n \lambda T}}{A} \quad (3.31)
\]

Calculating the sums in Equations 3-30 and 3-31 (see Problem 3-58) yields the

\[
\bar{E} = \frac{e^{\lambda T} - 1}{\lambda T - 1} = \frac{h\gamma}{\lambda T - 1} = \frac{hc}{\lambda T - 1} \quad (3.32)
\]

Multiplying this result by the number of oscillators per unit volume in the inter, we obtain for the energy density distribution function radiation in the cavity:

\[
(3.33) \quad u(\lambda) = \frac{8\pi \hbar c}{\lambda^5} \quad (\text{cm}^{-3})
\]

\[
(3.22) \quad f_n = A e^{-En/\lambda T} = A e^{\frac{-n\lambda T}{\lambda T}}
\]

\[
E_n = n\epsilon = n\hbar \lambda \quad \text{where} \quad n = \text{integer}
\]

\[
\epsilon = \text{quantized energy} \quad \lambda = \text{frequency} \quad \hbar = \text{Planck constant}
\]

\[
-l\frac{\epsilon}{\lambda T} - n\hbar \lambda /\lambda T - \frac{\lambda \lambda}{\lambda T}
\]

\[
\text{Let} \quad e = e = e \quad (x = \frac{\lambda \lambda}{\lambda T})
\]

Then \( \sum f_n = A \sum e^{-nx} = A e^{0} + e^{-x} + (e^{-x})^{2} + \cdots \int
\]

\[
\text{by definition}
\]

True: \( \text{Compare this to the series expansion for} \ (1-\gamma)^{-1} \)

\[
(1+\gamma)^{0} = 1 + \gamma^{2} + \frac{\gamma^{3}}{2^{2} - 2^{2} + \cdots}
\]

\[
(1-\gamma)^{-1} = 1 + \gamma + \gamma^{2} + \cdots
\]
Then $E_n = A(1-y)^{-1} = 1 \rightarrow A = $ 

\[ \sum_{n=0}^{\infty} E_n A e^{-E_n/kT} = \frac{1}{1-y} \sum_{n=0}^{\infty} e^{-n\hbar f/kT} = AhfZne^{-nx} \]

(3.31) $E = \sum_{n=0}^{\infty} E_n A e^{-E_n/kT} = \frac{1}{1-y} \sum_{n=0}^{\infty} e^{-n\hbar f/kT} = AhfZne^{-nx}$

Trick: $\frac{d}{dx} e^{-nx} = -nx e^{-nx}$ and we found $Ze^{-nx} = (1-y)^{-1}$

Combine these to get $Ze^{-nx} = -\frac{d}{dx} Ze^{-nx} = -\frac{d}{dx} (1-y)$

Evaluate in general and simplifying:

$-\frac{d}{dx} (1-y) = $ 

Then use $\frac{dy}{dx} = \frac{d}{dx} (e^{-x}) = $

Sub into (3.31) and show that $E = \frac{hf \gamma}{1-y}$.

Sub $m y = e^x$, multiply by $e^{-x}$, substitute in $x = \frac{hf \gamma}{kT}$ to get result.
The easy way: just use $E_n = -\frac{\hbar^2}{8m} R$ (4.23) and the reduced mass correction $\mu^2$

for the Rydberg constant: $R = R_\infty \left( \frac{1}{1 + \frac{\mu}{m}} \right) = \ldots$

Or, a real derivation of the energy level; using the reduced mass $\mu = m_1 m_2 / (m_1 + m_2)$ (as done in class!)

$$\frac{1}{2} \mu v^2 = \frac{ke^2}{r}$$

$$L = \mu v r$$

$$v = \ldots$$

Eliminate $v^2$ and solve for $r$

Substitute $r$ into (virial theorem?) $E_{\text{pot}} = \frac{1}{2} v = -\frac{1}{2} \frac{ke^2}{r}$

$$E_{\text{tot}} = E_n = \ldots$$

Either way, you should get half the energy of H atom.
\( E_1 = \) 

\[ E_2 = \frac{E_1}{2^2} \]

\[ E_3 = \frac{E_1}{3^2} \]

Lyman series lines are from transitions to the \( n=1 \) level.

Lyman \( \alpha \): \( E_2 \rightarrow E_1 \)

Lyman \( \beta \): \( E_3 \rightarrow E_1 \)