Ex 5.8 found by Ampere's law that
\[
\mathbf{B} = \mu_0 \mathbf{k} \times \mathbf{C} = \mu_0 \mathbf{e}_z \times \mathbf{A} = \mu_0 \mathbf{e}_z \times \left( \frac{\partial \mathbf{A}_x}{\partial y} - \frac{\partial \mathbf{A}_y}{\partial x} \right)
\]

\[
\mathbf{B}_x = \frac{\mu_0 \mathbf{k} \times \mathbf{B}}{2} = \frac{\partial A_y}{\partial z} - \frac{\partial A_z}{\partial y}
\]

If \( A_x = 0 \) then \( A_2 = \pm \frac{\mu_0 k}{2} \mathbf{x} \rightarrow \mathbf{A} = \frac{\mu_0 k}{2} \mathbf{x} + \text{constant} \)

or we could pick

\( A_2 = 0 \), then \( A_x = \pm \frac{\mu_0 k}{2} \mathbf{z} \rightarrow \mathbf{A} = -\frac{\mu_0 k}{2} \mathbf{z} + \mathbf{x} + \text{constant} \)

(These are not unique) \( A_x \) is more natural, since \( \mathbf{A} \) is parallel to \( \mathbf{k} \).
Problem 5.33 Show that the magnetic field of a dipole can be written in coordinate-free form:

\[ B_{\text{dip}}(r) = \frac{\mu_0}{4\pi} \frac{1}{r^3} (3(m \cdot \hat{r}) \hat{r} - m). \]  

\[(3.87)\]

\[ A_{\text{dip}}(r) = \frac{\mu_0 m \sin \theta}{4\pi r^2} \hat{\phi}. \]  

\[(5.85)\]

and hence

\[ B_{\text{dip}}(r) = \nabla \times A = \frac{\mu_0 m}{4\pi r^2} (2 \cos \theta \hat{r} + \sin \theta \hat{\phi}). \]  

\[(5.86)\]

The magnetic dipole moment

\[ \vec{m} = \frac{l}{\sqrt{a^2}} \]  

can be written

\[ \vec{m} = (\vec{m} \cdot \hat{r}) \hat{r} + (\vec{m} \cdot \hat{\phi}) \hat{\phi} \]

where \( \vec{m} \cdot \hat{r} = m \cos \theta \) and \( \vec{m} \cdot \hat{\phi} = msin \theta \)

\[ \vec{m} \]

\[ 3(\vec{m} \cdot \hat{r}) = 3m \cos \theta \]

\[ \text{So } \ 3(\vec{m} \cdot \hat{r}) \hat{r} - \vec{m} = 3m \cos \theta \hat{r} - (m \cos \theta \hat{r} + m \sin \theta \hat{\phi}) \]

\[ = 2m \cos \theta \hat{r} - m \sin \theta \hat{\phi} \]

Given in spherical coordinates (5.86):

\[ B = \frac{\mu_0 m}{4\pi r^3} (2 \cos \theta \hat{r} - msin \theta \hat{\phi}) \]

we can rewrite this in coordinate-free form as (5.87):

\[ B = \frac{\mu_0}{4\pi r^3} (3(\vec{m} \cdot \hat{r}) \hat{r} - \vec{m}) \]
Problems 5.38 It may have occurred to you that since parallel currents attract, the current within a single wire should contract into a tiny concentrated stream along the axis. Yet in practice the current typically distributes itself quite uniformly over the wire. How do you account for this? If the positive charges (density $\rho_+ \equiv \rho$) are at rest, and the negative charges (density $\rho_- \equiv \rho$) move at speed $v$ (and none of these depends on the distance from the axis), show that $\rho_- = -\rho + y^2$, where $y = 1/\sqrt{1 - (v/c)^2}$ and $c^2 = 1/\mu_0 e_0$. If the wire as a whole is neutral, where is the compensating charge located? [Notice that for typical velocities (see Prob. 5.19) the two charge densities are essentially unchanged by the current (since $y \approx 1$). In plasmas, however, where the positive charges are also free to move, this so-called pinch effect can be very significant.]

Excess charge carriers concentrate toward center... Excess + at near edges provide an attractive $E$ field to prevent further concentration at center.

Equilibrium: $f_B = f_e$. $q$\textbf{v} \times \textbf{B} = q$\textbf{v} \textbf{E} = $q$\textbf{E}

Find $B$: $\oint B \cdot dl = \mu_0 I$ \textbf{Cross} $\textbf{area}$ of $\text{cylindrical}$ $\textbf{wire}$, $I = \frac{V}{R}$

$B = \frac{\mu_0 I}{2\pi s}$

$B(s) = \frac{\mu_0 I}{2\pi s} = \frac{\mu_0 I}{2\pi s}$

Find $E$: $\oint E \cdot da = E \cdot 2\pi s \cdot I = \frac{q}{C_0}$ where $p = \frac{q}{\text{volume}}$, $p = p_1 + p_2$

$E(s) = \frac{p_1 - p_2}{\mu_0 2\pi s}$

$E(s) = \frac{p_1 - p_2}{\mu_0 2\pi s} = \frac{p_1 - p_2}{\mu_0 2\pi s}$

$p_1 - p_2 = \frac{\text{E}_0 \text{c}_0}{2 \pi s} \cdot p_1 - p_2 = p - \frac{\text{E}_0 \text{c}_0}{2 \pi s}$

$p_1 = p - (\frac{\text{E}_0 \text{c}_0}{2 \pi s})$
Problem 5.51 (The Hall effect.) A current $I$ flows to the right through a rectangular bar of conducting material, in the presence of a uniform magnetic field $B$ pointing out of the page (Fig. 5.59).

(a) If the moving charges are positive, in which direction are they deflected by the magnetic field?

This deflection results in an accumulation of charge on the upper and lower surfaces of the bar, which in turn produces an electrical force to counteract the magnetic one. Equilibrium occurs when the two exactly cancel.

(b) Find the resulting potential difference (the "Hall voltage") between the top and bottom of the bar, in terms of $B$, $v$ (the speed of the charges), and the relevant dimensions of the bar.

(c) How would your analysis change if the moving charges were negative? [The Hall effect is the classic way of determining the sign of the mobile charge carriers in a material.]

![Diagram of a current $I$ flowing through a rectangular bar with a magnetic field $B$ out of the page.]

Positive charges moving right would be deflected down by outward $B$.

$F = qE + q\vec{v} \times \vec{B} = 0$ when $\vec{E} = -\vec{v} \times \vec{B} = -\nabla \mathcal{V}$

$\mathcal{V} = -\int E \cdot dl = (\vec{v} \times \vec{B}) \cdot dl = vBt$ (higher voltage would be on the bottom, for moving $\Theta$)

Current due to negative charges moving left would also result in charges deflected down.

But this would make higher voltage on top. This is what is observed: charge carriers are negative (that is, they obey a left-hand rule).