1. Suppose that in a survey people are asked to rank the ice cream flavors chocolate, vanilla and strawberry in order from their first to last choice, with the results given below:

| | Percentage of Voters | | | | | |
|---|---|---|---|---|---|---|
| | 33 | 3 | 10 | 20 | 7 | 27 |
| Chocolate | 1 | 1 | 2 | 3 | 2 | 3 |
| Vanilla | 2 | 3 | 1 | 1 | 3 | 2 |
| Strawberry | 3 | 2 | 3 | 2 | 1 | 1 |

   (a) Which flavor wins using Borda count?

   (b) Which flavor wins using Plurality?

   (c) Which flavor wins using the Hare system?

   (d) Which flavor wins using sequential pairwise voting, with the agenda order Chocolate, Vanilla, Strawberry?

   (e) Is there a Condorcet winner?

2. Construct a voting preference list for three alternatives A,B,C in such a way that the Borda winner fails the Condorcet Winner Criterion.

3. Consider the following sequence of preference lists for candidates A,B,C and D.

| | Number of Votes | | | |
|---|---|---|---|---|
| | 7 | 6 | 5 | 3 |
| First | A | B | C | D |
| Second | B | A | B | C |
| Third | C | C | A | B |
| Fourth | D | D | D | A |

(a) Show that $A$ is the unique winner if the Hare system is used.

(b) Find the winner using the Hare system if all of the 3 voters in the last column put $A$ as their first choice (rather than their last choice), and all the other choices are pushed down by one in the rankings. Which of the four criteria for fair voting fails to hold here?

4. Consider the following sequence of preference lists for 4 voters.

| | Number of Votes | | |
|---|---|---|---|
| | 5 | 4 | 4 |
| First | A | B | C |
| Second | B | C | B |
| Third | C | A | A |

(a) Show that $A$ is the unique winner if Plurality or the Hare system is used.

(b) Show that with both systems if the voters in the last column change their preferences between the two losing candidates then $A$ is no longer the winner. Which of the fairness criteria does this violate?

(c) Does the change suggested above affect the outcome of the other two voting methods?