VISUALIZING RIEMANN SURFACES, TEICHMÜLLER SPACES, AND TRANSFORMATION GROUPS ON HYPERBOLIC MANIFOLDS USING REAL TIME INTERACTIVE COMPUTER ANIMATOR (RTICA) GRAPHICS

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Submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy in Mathematics
in the Graduate College of the
University of Illinois at Urbana-Champaign, 1998

Urbana, Illinois

Abstract

Garsia and Rüedy established the existence of conformal embeddings of all Riemann surfaces into $\mathbb{R}^3$. Pinkall constructed embeddings for all closed genus-one Riemann surfaces. In this work we present a new construction of embeddings of tori conformally equivalent to Euclidean rectangles. This construction does not rely on the Hopf fibration of S^3.

We present graphic tools of independent interest to geometers. The real-time interactive computer animators (RTICA) which incorporate these tools and which produce animated illustrations are described. In particular, we have RTICA that illustrate a Bishop and Frenet frame integrator and that illustrate the fundamental polyhedra in hyperbolic 3-space described by Jørgensen.

The production and presentation of RTICA are described as are the techniques for generating *pic* and *Mathematica* illustrations to be embedded into TEX documents.

Acknowledgement

I wish to thank the director of my research, George K. Francis, who provided inspiration and support throughout. The kindness offered by George and his wife Bettina will remain one of the jewels among my memories of graduate student life; a sense of gratitude could not be felt more deeply than is my own. The other members of my committee, Felix Albrecht, Richard Bishop, Horacio Porta, and Jack Wetzel, provided encouragement, for which I am deeply grateful. Others in this department, to whom I wish to extend a special thanks, are Jerry Uhl, Joe Miles, Bruce Reznick, James Langley, Eva Gray, Tony Peressini, Gerald Janusz, Stephanie Alexander, Julian Palmore, Leon McCulloh, Bruce Berndt, Monica Nicholau, Ralph Alexander, and John Sullivan. Outside of this department, I have benefited directly from the works and examples set by A. Marden, J. Quine, H. Ferguson, L. Keen, W. Abikoff, D. O'Shea, and M. Senechal.

In the same way that one can never truly repay one's debt to one's natural family, I do not expect to be able to repay debts incurred while a student. However, I wish to acknowledge and thank those who allowed me to incur these debts.

Dedication

To all of my family,

especially to Robert and Virginia.

Table of Contents