

Climate Change workshop #1

Estimate the number of stars in the observable universe

By Dr. Richard Gammon, UW Seattle

(recorded by Dr. E.J. Zita, Evergreen St. College, Olympia)

How many grains of sand on the Earth? Let's do an order of magnitude estimate, then compare with the number of stars. This is a warm-up exercise, to get comfortable with the powerful technique of order of magnitude estimation. We'll use our common sense, and scientific notation with about one significant figure, instead of relying on calculators with unreasonable precision. Make your own estimates if you like – ours are not written in stone.

First, estimate the size of a grain of sand. If its width is about $x \sim 0.5 \text{ mm} = 5 \times 10^{-4} \text{ m}$, then its volume $\sim x^3 = 125 \times 10^{-12} \text{ m}^3 \sim 10^{-10} \text{ m}^3 \sim V_{\text{grain of sand}}$

What is the volume of the Earth's beaches? If they average about $d = 3 \text{ m}$ deep and $w = 30 \text{ m}$ wide, then a beach cross section would be about $A = d * w \sim 100 \text{ m}^2$

What is the length of beaches on the planet? The US is about 3000 miles or 5000 km wide. Estimate 10,000 miles of beaches – more for curvier coasts, less for coasts without beach. Multiply by a few continents, and estimate $L = 100,000 \text{ km} = 10^8 \text{ m}$ of beaches on the planet.

Then the volume of beaches $V_{\text{beaches}} = L * A = 10^8 \text{ m} * 10^2 \text{ m}^2 = 10^{10} \text{ m}^3$.

To find the number of grains of sand N , note that $N * V_{\text{grain}} = V_{\text{beaches}}$, so $N = V_{\text{beaches}} / V_{\text{grain}} = 10^{10} / 10^{-10} = 10^{10-(-10)} = 10^{20}$ grains of sand.

Now compare this to the number of stars in the observable universe. There about 100 billion galaxies ($100 \times 10^9 = 10^{11}$), each with about 100 billion stars, for a total of $10^{11} \times 10^{11} = 10^{22}$ stars in the observable universe. This is close to the number of grains of sand in our estimate.

(Is that coincidental or causal?)

On the next page is a fill-in-the-blanks version suitable for use in classrooms.

Climate Change workshop #1

Estimate the number of stars in the observable universe

by Dr. Richard Gammon, UW Seattle, and Dr. E.J. Zita, Evergreen St. College, Olympia

How many grains of sand are on the Earth? Let's do an order of magnitude estimate, then compare with the number of stars in our universe. This is a warm-up exercise, to get comfortable with the powerful technique of order of magnitude estimation. We'll use our common sense, and scientific notation with about one significant figure, instead of relying on calculators with unreasonable precision.

0. How many stars are in the observable universe? Here is an example of using scientific notation to easily estimate huge quantities without a calculator.

There about 100 billion galaxies: $100 \times 10^9 = 10^2 \times 10^9 = 10^{2+9} = 10^{11} \sim N_{\text{galaxies}}$.

Each galaxy has about 100 billion stars: $N_{\text{stars per galaxy}} \sim 10^{11}$.

This gives a total of $N_{\text{stars per galaxy}} * N_{\text{galaxies}} = 10^{11} \times 10^{11} = 10^{11+11} = \underline{10^{22}}$ stars in the observable universe.

1. First, estimate the size of a grain of sand in mm: $x = \underline{\hspace{2cm}}$ mm

Now convert this to meters in scientific notation,

using the fact that $1 \text{ mm} = 10^{-3} \text{ m}$: $x = \underline{\hspace{1cm}} \times 10^{\underline{\hspace{1cm}}} \text{ m}$

The grain's volume will be approximately $V_{\text{grain}} \sim x^3 = \underline{\hspace{1cm}} \times 10^{\underline{\hspace{1cm}}} \text{ m}^3$
Estimate this without using a calculator!

Compare your estimate for V_{grain} to those of classmates. You should have slightly *different* volumes V_{grain} , if you started with slightly different sizes x , and they should be roughly in the same ballpark. Discuss the estimation process with your teacher.

2. Next, let's find the volume of Earth's beaches. First common sense, no calculations:

a. Estimate the depth of a typical beach in meters: $d = \underline{\hspace{2cm}}$ m deep

b. Estimate the width of a beach, from the ocean to where the sand meets land or rock:
 $w = \underline{\hspace{2cm}}$ m wide

c. Now calculate the product $d * w = \underline{\hspace{1cm}} \times 10^{\underline{\hspace{1cm}}} \text{ m}^2 = A$

The power of 10 is more important than the factor in front. Let's just say $A \sim 10^{\underline{\hspace{1cm}}} \text{ m}^2$

d. What is the volume of beaches on the planet? You could make this estimate in a few steps. Here is one method – if you have a better one, go for it.

i. How wide is the US? _____ miles or _____ km wide.

ii. By what factor should you multiply this to get the length of coastline with beaches on North America? ____ Estimate the total length of beaches on North America:

$$L_{NA} = \text{_____ km} = 10 \text{ _____ m}$$

iii. How many continents have beaches? By what factor should you multiply this to get the length of beaches on Earth? ____ Estimate the total length of beaches on Earth:

$$L = 10 \text{ _____ m}$$

iv. Then the volume of beaches $V_{\text{beaches}} = L * A = \text{_____ m} * \text{_____ m}^2 = \text{_____ m}^3$.

3. To find the number of grains of sand N, note that $N * V_{\text{grain}} = V_{\text{beaches}}$, so

$$N = V_{\text{beaches}} / V_{\text{grain}} = 10 \text{ _____} / 10 \text{ _____} = 10 \text{ _____} - (\text{_____}) = 10 \text{ _____ grains of sand.}$$

4. Now compare this to the number of stars in the observable universe. We found that there about 100 billion galaxies ($100 \times 10^9 = 10^{11}$), each with about 100 billion stars, for a total of $10^{11} \times 10^{11} = 10^{22}$ stars in the observable universe.

How close is this to the number of grains of sand in your estimate?

If it's close, is there a reason, or is it just coincidence? Discuss this with your class.