E17 from Foukal Ch7. Find the dispersion relation for p-modes resonant beneath the photosphere

1 July 04
Boulder

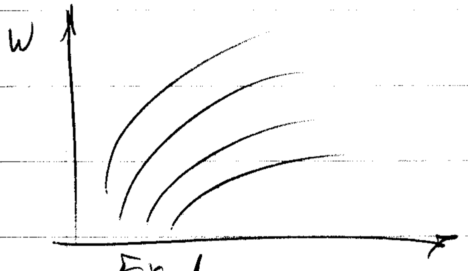


Fig. 1

observed. (Foukal p. 227)

→ Guess a relationship between ω & k:

$$k^{\square} \propto \omega^{\square}$$

Model a resonant cavity of depth δ with boundary conditions

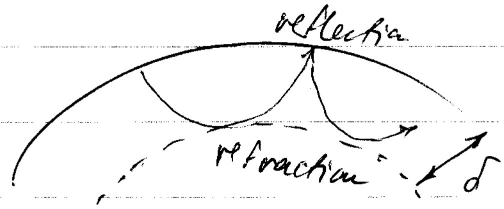
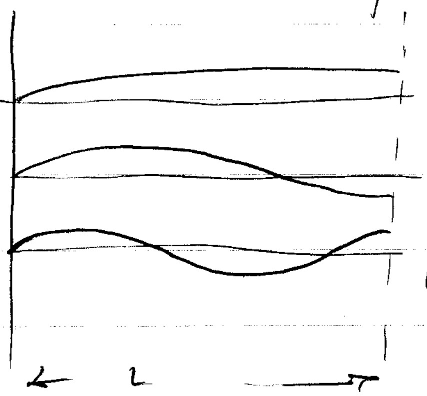


Fig. 2

reflection closed

refraction open

Fig. 3



$L = \frac{1}{4}\lambda$

$L = \_\_\,\lambda$

$L = \_\_\,\lambda$

(a)

General relationship between L & λ:

$$L = \frac{m\lambda}{\square} \quad \text{where}$$

(b) $m = \_\_\_\_\_\_$

(a) $4L = m\lambda$: Let $k = \frac{2\pi}{\lambda}$ and $\frac{m}{2} = (n + \frac{1}{2})$

odd   integer 0, 1, 2, ...

$(n+\frac{1}{2})$

Solve for kL

(1) depth of resonant cavity $\delta = L =$

To get a relationship between k and ω, next find a relationship between δ and ω, where $\delta = L$ = depth of resonant cavity.

(2) Sound speed $c_s^2 = \frac{\gamma P}{\rho} = \frac{\omega^2}{k^2}$ / P = pressure, ρ = density; ω = freq, k = wavenumber

Hydrostatic equilibrium (HSE): $\nabla P = \frac{\partial P}{\partial z} = g\rho$ (3)

pressure gradient, gravity

Constant-entropy equation of state: $P = a\rho^{\gamma}$ (4)

a = some constant, $\gamma = \frac{c_p}{c_v}$

Solve (3) and (4) for $\rho^{\gamma-1}$: $g\rho = \frac{d}{dz}\left(a\rho^{\gamma}\right)$ (5)

(d) Hint: Let $y = \rho^{\gamma}$. Then dy = ____________ $d\rho$

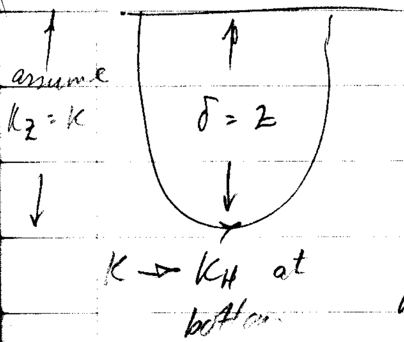
Separate variables: ____ dz = ____________ $d\rho$

(e) Integrate: $\int \frac{g}{a}\, dz$ = ________________

(6) $\rho^{\gamma-1} =$

(f) Now sub (4) and (6) into (2) and simplify.

(7) $c_s^2 = \qquad\qquad = \dfrac{\omega^2}{k^2}$



At the base of the resonant cavity, the sound wave (p-mode) of speed c_s has a purely horizontal wave number.

(g) Let $z = \delta$ and $k = k_H$ and solve for δ

(f) Finally, sub into (1) and solve for ω^2:

How does this analytic $\omega(k)$ compare to the form of $\omega(k)$ you guessed for the data in Fig. 1?

What assumptions have we made? Discuss.